DISCUSSION

ON THE PAPER OF RUTH B. MARCUS *

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Prof. Marcus: We seem still at the impasse I thought to resolve at this time. The argument concerning (12) was informal, and parallels as I suggested, questions raised in connection with the 'paradox' of analysis. One would expect that if a statement were analytic, and it bore a strong equivalence relation to a second statement, the latter would be analytic as well. Since (12) cannot be represented in S_m without restriction, the argument reveals material equivalence to be insufficient and weak. An adequate representation of (12) requires a modal framework.

The question I have about essentialism is this: Suppose these modal systems are extended in the manner of *Principia* to higher orders.

Then

$$\Box\,((5 + 4) = 9)$$

is a theorem ('$=$' here may be taken as either '$=_{\mathrm{s}}$' or '$=_{\mathrm{m}}$' of the present paper, since the reiterated squares telescope), whereas

$$\Box\,((5 + 4) = \text{the number of planets})$$

is not. Our interpretation of these results commits us only to the conclusion that the equivalence relation which holds between $5 + 4$ and 9 is stronger than the one which holds between $5 + 4$ and the number of planets. More specifically, the stronger one is the class or attribute analogue of $\equiv$. No mysterious property is being conferred on either 9 or the number of planets which it doesn't already have in the extensional $((5 + 4) =_{\mathrm{m}}$ the number of planets).

Prof. Quine: May I ask if Kripke has an answer to this?... Or I'll answer, or try to.

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Mr. Kripke: As I understand Professor Quine's essentialism, it isn't what's involved in either of these two things you wrote on the board, that causes trouble. It is in inferring that there exists an x, which necessarily $= 5 + 4$ (from the first of the two). (to Quine:) Isn't that what's at issue?

Prof. Quine: Yes.

Mr. Kripke: So this attributes necessarily equalling $5 + 4$ to an object.

Prof. Marcus: But that depends on the suggested interpretation of quantification. We prefer a reading that is not in accordance with things, unless, as in the first order language there are other reasons for reading in accordance with things.

Prof. Quine: That's true.

Prof. Marcus: So the question of essentialism arises only on your reading of quantification. For you, the notion of reference is univocal, absolute, and bound up with the expressions, of whatever level, on which quantification is allowed. What I am suggesting is a point of view which is not new to the history of philosophy and logic. That all terms may refer to objects, but that not all objects are things where a thing is at least that about which it is appropriate to assert the identity relation. We note a certain historical consistency here, as for example, the reluctance to allow identity as a relation proper to propositions. If one wishes, one could say that object-reference (in terms of quantification) is a wider notion than thing-reference, the latter being also bound up with identity and perhaps with other restrictions as well such as spatio-temporal location. If one wishes to use the word 'refer' exclusively for thing-reference, then we would distinguish those names which refer, from those which name other sorts of objects. Considered in terms of the semantical construction proposed at the end of the paper, identity is a relation which holds between individuals; and their names have thing-reference. To say of a thing a that it necessarily has a property φ ($\Box$ (φa)), is to say that $\varphi\ a$ is true in every model. Self-identity would be such a property.

Prof. Quine: Speaking of the objects or the referential end of things in terms of identity, rather than quantification, is agreeable to me in the sense that for me these are inter-definable anyway. But what's appropriately regarded as the identity matrix, or open sentence, in the theory is for me determined certainly by consideration of quantification. Quantification is a little bit broader, a little bit more generally applicable to the theory because you don't always have anything that would fulfill this identity requirement. As to where essentialism comes in: what I have in mind is an interpretation of this quantification where you have an x here (in $\Box((5 + 4) = x)$). Now I appreciate that from the point of view of modal logic, and of things that have been done in modal logic in Professor Marcus' pioneer system, this would be regarded as true rather than false:

$$\Box((5 + 4) = 9)$$

This is my point, in spite of the fact that if you think of this ($\Box$ (5 + 4) = the number of planets) as what it is generalized from, it ought to be false.

Prof. Marcus: $\Box$ ((5 + 4) = the number of planets) *would* be false. But this does not preclude the truth of

$$(\exists x) \Box (5 + 4) = x) =$$

anymore than the falsehood

$$12 = \text{the number of Christ's disciples}$$

precludes the truth of

$$(\exists x)(12 = x).$$

(We would, of course, take '$=$' as '$=_m$' here.)

Prof. Quine: That's if we use quantification in the ordinary ontological way and that's why I say we put a premium on the *nine* as over against the *number of planets*; we say this term is what is going to be *maßgebend* for the truth value of this sentence in spite of the fact that we get the opposite whenever we consider the other term. This is the sort of specification of the number that counts:

$$5 + 4 = 9$$

This is not: $5 + 4 =$ number of planets.

I grant further that essentialism does not come in if we interpret quantification in your new way. By quantification I mean, quantification in the ordinary sense rather than a new interpretation that might fit most if not all of the formal laws that the old quantification fits. I say 'if not all', because I think of the example of real numbers again. If on the other hand we do not have quantification in the old sense then I have nothing to suggest at this point about the ontological implications or difficulties of modal logic. The question of ontology wouldn't arise if there were no quantification of the ordinary sort. Furthermore, essentialism certainly wouldn't be to the point, for the essentialism I'm talking about is essentialism in the sense that talks about objects, certain objects; that an object has certain of these attributes essentially, certain others only accidentally. And no such question of essentialism arises if we are only talking of the terms and not the objects that they allegedly refer to. Now Professor Marcus also suggested that possibly the interpretation could be made something of a hybrid between the two – between quantification thought of as a formal matter, and just talking in a manner whose truth conditions are set up in terms of the expression substituted rather than in terms of the objects talked about; and that there are other cases where we can still give quantification the same old force. Now that may well be: we might find that in the ordinary sense of quantification I've been talking about there is quantification into non-modal contexts and no quantification but only this sort of quasi-quantification into the modal ones. And this conceivably might be as good a way of handling such modal matters as any.

Prof. Marcus: It is not merely a way of coping with perplexities associated with intensional contexts. I think of it as a better way of handling quantification.

You've raised a problem which has to do with the real numbers. Perhaps the Cantorian assumption is one we can abandon. We need not be particularly concerned with it here.

Prof. Quine: It's one thing I would certainly be glad to avoid, if we can get all of the classical mathematics that we do want.

Mr. Kripke: This is what I thought the issue conceivably might be, and hence I'll raise it explicitly in this form: Suppose this system contains names, and suppose the variables are supposed to range over numbers, and using "9" as the name of the number of planets, and the usual stock of numerals, "0", "1", "2", , , and in addition various other primitive terms for numbers, one of which would be "NP" for the "number of planets", and suppose "$\Box$ (9 $>$ 7)" is true, according to our system. But say we also have "$\sim\Box$ (NP $>$ 7)". Now suppose "NP" is taken to be as legitimate a name for the number of planets as "9", (i.e. for this *number*) as the numeral itself. Then we get the odd seeming conclusion, (anyway in your (Marcus') quantification) that

$$(\exists\, x, y)\left(x = y \cdot \Box\,(x > 7) \cdot \sim\,\Box\,(y > 7)\right)$$

On the other hand, if "*NP*" is not taken to be as legitimate a name for the number of planets as "9", then, in that case, I presume that Quine would reply that this sort of distinction amounts to the distinction of essentialism itself. (To Quine:) Would this be a good way of stating your position?

Prof. Quine: Yes. And I think this formula is one that Professor Marcus would accept under a new version of quantification. Is that right?

Prof. Marcus: No... this wouldn't be true under my interpretation, if the '$=$' (of Kripke's expression) is taken as identity. If it were taken as identity, it would be not only odd-seeming but contradictory. If it is taken as '$=_{\mathrm{m}}$' then it is not odd-seeming but true. What we must be clear about is that in the extended modal systems with which we are dealing here, we are working within the framework of the theory of types. On the level of individuals, we have only identity as an equivalence relation. On the level of predicates, or attributes, or classes, or propositions, there are other equivalence relations which are weaker. Now the misleading aspect of your (Kripke's) formulation is that when you say, "let the variables range over the numbers", we seem to be talking about individual variables, '$=$' must then name the identity relation and we are in a quandry. But within a type framework, if x and y can be replaced by names of numbers, then they are higher type variables and the weaker equivalence relations are appropriate in such contexts.

Mr. Kripke: Well, you're presupposing something like the Frege-Russell definition of number, then?

Prof. Marcus: All right. Suppose numbers are generated as in *Principia* and suppose 'the number of planets' may be properly equated with '9'. The precise nature of this equivalence will of course depend on whether 'the number of planets' is interpreted as a description or a predicate, but in any case, it will be a weak equivalence.

Mr. Kripke: Nine and the number of planets do not in fact turn out to be identically the same?

Prof. Marcus: No, they're not. That's just the point.

Mr. Kripke: Now, do you admit the notion of 'identically the same' at all?

Prof. Marcus: That's a different question. I admit identity on the level of individuals certainly. Nor do I foresee any difficulty in allowing the identity relation to hold for objects named by higher type expressions (except perhaps propositional expressions), other than the ontological consequences discussed in the paper. What I am *not* admitting is that 'identically the same' is indistinguishable from weaker forms of equivalence. It is explicit or implicit extensionalizing principles which obliterate the distinction. On this analysis, we could assert that

9 is identically the same as 9

but not

9 is identically the same as $(5 + 4)$

without some weak extensionalizing principle which reduces identity to logical equivalence.

Mr. Kripke: Supposing you have any identity, and you have something varying over individuals.

Prof. Marcus: In the theory of types, numbers are values for predicate variables of a kind to which several equivalence relations are proper.

Mr. Kripke: Then, in your opinion the use of *numbers* (rather than individuals) in my example is very important.

Prof. Marcus: It's crucial.

Prof. Quine: That's what I used to think before I discovered the error in Church's criticism. And if I understand you, you're suggesting now what I used to think was necessary; namely, in order to set these things up, we're going to have, as the values of variables, not numbers, but assorted number properties, that are equal, but different numbers – the number of planets on the one hand, 9 on the other. What I say now is that this proliferation of entities isn't going to work. For example, take x as just as narrow and intensional an object as you like...

Prof. Marcus: Yes, but not on the level of individuals where only one equivalence relation is present. (We are omitting here consideration of such relations as congruence.)

Prof. Quine: No, my x isn't an individual. The values of 'x' may be properties, or attributes, or propositions, that is as intensional as you like. I argue that if $\varphi(x)$ determines x uniquely, and if p is not implied by $\varphi(x)$, still the conjunction $p \cdot \varphi(x)$ will determine that same highly abstract attribute, or whatever it was, uniquely, and yet these two conditions will not be equivalent, and therefore this kind of argument can be repeated for it. My point is, we can't get out of the difficulty by splitting up the entities; we're going to have to get out of it by essentialism. I think essentialism, from the point of view of the modal logician, is something that ought to be welcome. I don't take this as being a *reductio ad absurdum.*

Prof. McCarthy: (*MIT*) It seems to me you can't get out of the difficulty by making 9 come out to be a class. Even if you admit your individuals to be much more inclusive than numbers. For example, if you let them be truth values. Suppose you take the truth value of the 'number of planets is nine', then this is something which is true, which has the value truth. But you would be in exactly the same situation here. If you carry out the same problem, you will still get something which will be 'there exists x, y such that $x = y$ and it is necessary that x is true, but it is not necessary that y is true'.

Prof. Marcus: In the type framework, the individuals are neither numbers, nor truth values, nor any object named by higher type expressions. Nor are the values of sentential variables truth values. Sentential or propositional variables take as values sentences (statements, names of propositions if you will). As for your example, there is no paradox since your '$=$' would be a material equivalence, and by virtue of the substitution theorem, we could not replace 'y' by 'x' in '$\Box\ x$' (x being contingently true).

Prof. McCarthy: Then you don't have to split up numbers, regarding them as predicates either, unless you also regard truth functions as predicates.

Prof. Marcus: About "splitting up". If we must talk about objects, then we could say that the objects in the domain of individuals are extensions, and the objects named by higher order expressions are intensions. If one is going to classify objects in terms of the intension – extension dualism, then this is the better way of doing it. It appears to me that a failing of the Carnap approach to such questions and one which generated some of these difficulties, is the passion for symmetry. Every term (or name) must, according to Carnap, have a dual role. To me it seems unnecessary and does proliferate entities unnecessarily. The kind of evidence relevant here is informal. We do, for example, have a certain hesitation about talking of identity of propositions and we do acknowledge a certain difference between talking of identity of attributes as against identity in connection with individuals. And to speak of the intension named by a proper name strikes one immediately as a distortion for the sake of symmetry.

Follesdal: The main question I have to ask relates to your argument against Quine's examples about mathematicians and cyclists. You say that (55) is not provable in *QS*4. Is your answer to Quine that it is not provable?

Prof. Marcus: No. My answer to Quine is that I know of no modal system, extended of course, to include the truth of:

It is necessary that mathematicians are rational

and

It is necessary that cyclists are two-legged

by virtue of meaning postulates or some such, where his argument applies. Surely if the argument was intended as a criticism of modal logic, as it seems to be, he must have had *some* formalization in mind, in which such paradoxes might arise.

Follesdal: It seems to me that the question is not whether the formula is provable, but whether it's a well-formed formula, and whether it's meaningful.

Prof. Marcus: The formula in question is entirely meaningful, well-formed if you like, given appropriate meaning postulates (defining statements) which entail:

All mathematicians are rational

and

All cyclists are two-legged .

I merely indicated that there would be no way of *deriving* from these meaning postulates (or defining statements) as embedded in a modal logic, anything like:

It is necessary that John is rational

given the truth:

John is a mathematician

although both statements are well-formed and the relation between 'mathematician' and 'rational' is analytic. The paradox simply does not arise. What I *did* say is that there is a derivative sense in which one can talk about necessary attributes, in the way that abstraction is derivative.

For example, since it is true that

$$(x) \Box (xIx)$$

which with abstraction gives us

$$(x) \Box (x \epsilon \hat{y}(yIy))$$

which, as we said before, would give us

$$\vdash \boxdot \hat{y}(yIy)$$

The property of self-identity may be said to be necessary, for it corresponds to a tautological function. Returning now to Professor Quine's example, if we introduced constants like 'cyclist', 'mathematician', etc., and appropriate meaning postulates then the attribute of being either a non-mathematician or rational, would also be necessary. Necessary attributes would correspond to analytic functions in the broader sense of analytic. These may be thought of as a kind of essential attribute, although necessary attribute is better here. For these are attributes which belong necessarily to every object in the domain whereas the usual meaning of essentialism is more restricted. Attributes like mathematician and cyclist do not correspond to analytic functions.

Prof. Quine: I've never said or, I'm sure, written that essentialism could be proved in any system of modal logic whatever. I've never even meant to suggest that any modal logician even was aware of the essentialism he was committing himself to, even implicitly in the sense of putting it into his axioms. I'm talking about quite another thing – I'm not talking about theorems, I'm talking about truth, I'm talking about true interpretation. And what I have been arguing is that if one is to quantify into modal contexts and one is to interpret these modal contexts in the ordinary modal way and one is to interpret quantification as quantification, not in some quasi-quantificatory way that puts the truth conditions in terms of substitutions of expressions, – then in order to get a coherent interpretation one has got to adopt essentialism, and I already explained a while ago just how that comes about. But I did not say that it could ever be deduced in any of the S-systems or any system I've ever seen.

Prof. Marcus: I was not suggesting that you contended that essentialism could be *proved* in any system of modal logic. But only that I know of no interpreted modal system, even where extended to include predicate constants such as those of your examples, where properties like being a mathematician would necessarily belong to any object. The kind of uses to which *logical* modalities are put have nothing to do with essential properties in the old ontological sense. The introduction of physical modalities would bring us closer to this sort of essentialism.

Follesdal: That's what creates the trouble when one thinks about properties of this kind, like being a cyclist.

Prof. Quine: But then we can't use quantifiers as quantifiers.

Prof. Marcus: The interpretation of quantification has advantages other than those in connection with modalities. For example, many of the perplexities in connection with quantification raised by Strawson in *Introduction to Logical Theory* are clarified by the proposed reading of quantification. Nor is it my conception. One has only to turn to the Introduction of *Principia Mathematica* where existential quantification is discussed in terms of 'always true' and 'sometimes true'. It is a way of looking at quantification that has been neglected. Its neglect is a consequence of the absence of a uniform, colloquial way of translating although we can always find some adequate locution in different classes of cases. It is *easier* to say 'There is a thing which...' and since it is adequate some of the time it has come to be used universally with unfortunate consequences.

Prof. Quine: Well, Frege, who started quantification theory, had the regular ontological interpretation. Whitehead and Russell fouled it up because they confused use and mention.

Follesdal: It seems from the semantical considerations that you have at the end of the paper, that you need your special axiom.

Prof. Marcus: Yes, for that construction. I have no strong preferences. It would depend on the uses to which some particular modal system is to be put.

Follesdal: You think you might have other constructions?

Prof. Marcus: Indeed. Kripke, for example, has suggested other constructions. My use of this particular construction is to suggest that in discussions of the kind we are having here today, and in connection with the type of criticism raised by Professor Quine in *Word and Object* and elsewhere, it is perhaps best carried out with respect to some construction.

Mr. Kripke: Forgetting the example of numbers, and using your interpretation of quantification – (there's nothing seriously wrong with it at all) – does it not require that for any two names, 'A' and 'B', of individuals, 'A = B' should be *necessary*, if true at all? And if 'A' and 'B' are names of the same individual, that any necessary statement containing 'A' should remain necessary if 'A' is replaced by 'B'?

Prof. Marcus: We might want to say that for the sake of clarity and ease of communication that it would be convenient if to each object there were attached a single name. But we can and we do attach more than one name to a single object. We are here talking of proper names in the ideal sense, as tags and not descriptions. Presumably, if a single object had more than one tag, there would be a way of finding out such as having recourse to a dictionary or some analogous inquiry, which would resolve the question as to whether the two tags denote the same thing. If 'Evening Star' and 'Morning Star' are considered to be two proper names for Venus, then finding out that they name the same thing as 'Venus' names is different from finding out what is Venus' mass, or its orbit. It is perhaps admirably flexible, but also very confusing to obliterate the distinction between such linguistic and properly empirical procedures.

Mr. Kripke: That seems to me like a perfectly valid point of view. It seems to me the only thing Professor Quine would be able to say and therefore what he must say, I hope, is that the assumption of a distinction between tags and empirical descriptions, such that the truth-values of identity statements between tags (but not between descriptions) are ascertainable merely by recourse to a dictionary, amounts to essentialism itself. The tags are the "essential" denoting phrases for individuals, but empirical descriptions are not, and thus we look to statements containing "tags", not descriptions, to ascertain the essential properties of individuals. Thus the assumption of a distinction between "names" and "descriptions" is equivalent to essentialism.

Prof. Quine: My answer is that this kind of consideration is not relevant to the problem of essentialism because one doesn't ever need descriptions or proper names. If you have notations consisting of simply propositional functions (that is to say predicates) and quantifiable variables and truth functions, the whole problem remains. The distinction between proper names and descriptions is a red herring. So are the tags. (Marcus: Oh, no.)

All it is is a question of open sentences which uniquely determine. We can get this trouble every time as I proved with my completely general argument of p in conjunction with φx where x can be as finely discriminated an intension as one pleases – and in this there's no singular term at all except the quantifiable variables or pronouns themselves. This was my answer to Smullyan years ago, and it seems to me the answer now.

Mr. Kripke: Yes, but you have to allow the writer what she herself says, you see, rather than arguing from the point of view of your own interpretation of the quantifiers.

Prof. Quine: But that changes the subject, doesn't it? I think there are many ways you can interpret modal logic. I think it's been done. Prior has tried it in terms of time and one thing and another. I think any consistent system can be found an intelligible interpretation. What I've been talking about is quantifying, in the quantificational sense of quantification, into modal contexts in a modal sense of modality.

Mr. Kripke: Suppose the assumption in question is right – that every object is associated with a tag, which is either unique or unique up to the fact that substituting one for the other does not change necessities, – is that correct? Now then granted this, why not read "there exists an x such that necessarily p of x" as (put in an ontological way if you like) "there exists an object x with a name a such that p of a is analytic." Once we have this notion of name, it seems unexceptionable.

Prof. Quine: It's not very far from the thing I was urging about certain ways of specifying these objects being by essential attributes and that's the role that you're making your attributes play.

Mr. Kripke: So, as I was saying, such an assumption of names is equivalent to essentialism.

Prof. Cohen: I think this is a good friendly note on which to stop.