

# Propositional Logic

---

## Language $L_1$

### BASIC SYMBOLS:

- i. Sentence letters:  $P, \dots, Z$   
(including numerical subscripts, e.g.  $P_4, Q_{23}$ , etc.)
- ii. Connectives:  $\neg, \rightarrow$
- iii. Punctuation:  $), ($

### FORMATION RULES:

1. Any sentence letter is a sentence,
2. If  $\phi$  is a sentence, then  $\neg\psi$  is a sentence,
3. If  $\phi$  and  $\psi$  are a sentences, then  $(\phi \rightarrow \psi)$  is a sentence,
4. And nothing else is a sentence of  $L_1$  unless it can be constructed by means of these rules.

## Basic Inference Rules

$$\frac{\begin{array}{l} (\phi \rightarrow \psi) \\ \phi \end{array}}{\psi} \quad \text{modus ponens (mp)}$$

$$\frac{\begin{array}{l} \phi \\ \neg\neg\phi \end{array}}{\phi} \quad \text{double negation (dn)}$$

$$\frac{\begin{array}{l} (\phi \rightarrow \psi) \\ \neg\psi \end{array}}{\neg\phi} \quad \text{modus tollens (mt)}$$

$$\frac{\phi}{\phi} \quad \text{repetition (r)}$$

---

*Example derivation.*

$$(P \rightarrow \neg Q). (Z \rightarrow X). (\neg Z \rightarrow Q). \neg X \quad \therefore \neg P$$

1.	Show $\neg P$	
2.	$Z \rightarrow X$	pr2
3.	$\neg X$	pr4
4.	$\neg Z$	23mt
5.	$\neg Z \rightarrow Q$	pr3
6.	$Q$	45mp
7.	$\neg\neg Q$	6 dn
8.	$P \rightarrow \neg Q$	pr1
9.	$\neg P$	78mt
10.		9 dd