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Special issue in honour of Landon Rabern

Gottlob Frege memorably observed that every good mathematician is at least half a philosopher, and every good philosopher is at least half a mathematician [15]. Landon Rabern was a gifted mathematician who embodied Frege's ideal. He approached difficult mathematical problems but always with one eye on the most wide-reaching implications, and broader philosophical connections. As one collaborator commented,

> "It seemed like he was looking for some kind of miraculous, universal theorem. We called it The All Things Theorem. This theorem was a myth, but it represented Landon's approach very well. If there were such a theorem, he was on a mission to find it." [37]

Landon's work spanned a wide range of topics, including graph coloring, logic, and artificial intelligence. He was also a creative and generous colleague who shared his ideas and insights with many researchers around the world. His sudden death on October 19, 2020, at the age of 39, was a great loss to those close to him, as well as the mathematical community. This special issue is dedicated to his memory, as a tribute to his many research achievements. It contains 10 new articles written by his collaborators, friends, and colleagues that showcase his interests.

Landon was born and raised in Roseburg, Oregon, a small town known historically for its lumber industry. Despite growing up with limited financial resources and no particular academic focus in his family, Landon displayed a natural talent for learning from a young age. In addition to participating in typical childhood activities like sports and video games, he taught himself advanced topics such as number theory and computer programming. Landon became passionate about machine intelligence and developed a chess engine called "Betsy" during high school that was credited as the first published engine able to play Fischer Random Chess. In his free time, he and his friends built large Tesla coils in the garage that produced impressive lightning bolts.

Landon's formal education in higher mathematics began at Washington University in St. Louis, where he earned his bachelor's degree in 2003, while completing a computer science minor, publishing an undergraduate paper [20] and sharing the Ross Middlemiss Prize, awarded annually to the strongest mathematics major. He then enrolled in the doctoral program in mathematics at the University of California, Santa Barbara, where he was a teaching assistant majoring in algebra. By 2005, he had completed a Master's degree in mathematics, and all work for a doctoral degree, except for one class and a dissertation. Until this point his academic career had been excellent, but standard.

In 2005, Landon realized that he was not dependent on an academic position to support his mathematical research; rather, he could rely on his computer skills for a living, while pursuing his true mathematical interests. From 2005–2011 he worked as a programmer-software architect for various artificial intelligence and social media companies, while co-founding LBD Data and serving as its chief technical officer. At the same time, after completing a paper in algebra [22], he embarked on a program of research in graph theory, a subject that was not available to him at UCSB. As a side benefit of this plan, he was able to take advantage of powerful, industrial computers during their off-peak hours. He taught himself graph theory, while reading widely on the subject, and established a goal of proving the Borodin–Kostochka conjecture.

Conjecture 1 *(Borodin and Kostochka [3]). Every graph G with maximum degree $\Delta(G)$ at least $b := 9$ has chromatic number $\chi(G) \leq \max\{\Delta(G) - 1, \omega(G)\}$.*

This conjecture strengthens Brooks' Theorem for graphs G with $\Delta(G) \geq b$. Reed [36] had already used probabilistic methods to prove this conjecture for $b = 10^{14}$, but Landon wanted to complete the proof by determining the least $b \geq 9$ for which the conjecture is true. This involves hunting for counterexamples with $\Delta < b$, while trying to show that there are none with $\Delta = b$. With coauthors, Landon extended Reed's result to list coloring for graphs with sufficiently large maximum

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degree [5, this issue]. One of Landon's strategies was to use automated theorem proving to obtain partial results, and then prove these results by hand with the goal of finding more general proof techniques [10,13,14]. Another strategy was to study and tweak all existing proofs of Brooks' Theorem, while experimenting with new and simpler proofs [31], [26], [8]. In the current volume we have included yet another of his proofs that notably never mentions connectivity. By 2011, he had published five graph theory papers [21,23–26] and had submitted nine more [27–29,7,19,31,30,32,9].

In 2011 Landon entered the doctoral program in Mathematics at Arizona State University. He was given credit for 30 hours of coursework and graduate exams, but was required to pass a comprehensive examination in graph theory, complete 54 credit hours and, of course, write a dissertation. Upon arrival he immediately passed the comprehensive. Over the next two years, he completed the required credit hours, while polishing and expanding his previous research. He completed his Ph.D. in mathematics in 2013 under the supervision of Hal Kierstead with an excellent dissertation entitled "Coloring graphs from almost maximum degree sized palettes". A senior computer scientist on Landon's doctoral committee was so impressed by Landon's defense that he suggested that ASU should immediately hire Landon with tenure.

Although Landon's chief research aim was proving the Borodin–Kostochka conjecture, he worked much more broadly within graph coloring. With the help of his collaborators, Landon wrote numerous papers that inspired significant follow-up work by others and that still impact the field today. These include his work on hitting sets [25], the Hilton–Zhao conjecture [14], list critical graphs [18], and Reed's Conjecture [24]. His survey "Brooks' Theorem and Beyond" [8] elicited this praise from a reviewer: "...would also be a fine subject for a seminar on graph coloring, and would be equally at home in a course on mathematical writing. It is exceptionally well written..."

Landon was clearly a talented mathematician, but he was also an excellent programmer. So he embodied not just Frege's ideal (mentioned above) but also a modern riff on Frege's ideal, since he was at least half a computer scientist. Beyond his academic work, Landon also made significant contributions to industry, co-founding his own software company and working as a software engineer and data scientist at artificial intelligence and social media companies. His work in industry was often mathematical, such as the abstract theory he developed for merging in user-submitted edits to a map.

Landon particularly enjoyed applying his programming skills to problems in graph coloring, especially those in the vicinity of Reed's conjecture ([25]). In this direction, he also collaborated with Dieter Gernert and helped develop the KBGRAPH project. This was a knowledge-based system for computer-assisted proofs in graph theory started in 1985. They demonstrated how the technology could be applied to get partial results on Reed's conjecture ([17], [16]). Landon didn't continue to work on this particular system, but throughout his career he did continue to explore ways of employing computer-assistance for mathematical research (e.g., see WebGraphs [34] and SageGraphUI [33]). A nice example of his creative use of computer-assistance occurs in his work with Daniel Cranston where he programmed the computer to write a proof as a "choose-your-own adventure" – they showed that regardless of what happened in the graph, the story always ended the same way: the proof succeeded ([11], [12]).

Landon's work on logic included a paper on 'the hardest logic puzzle ever'. This is a famous puzzle of Raymond Smullyan (via [2]). While a graduate student he and his brother published an article in the journal *Analysis*—the preeminent and highly regarded periodical for concise philosophical works. Among clarifying some ambiguities in the puzzle concerning randomness, they showed how to solve the puzzle in only two questions by using paradoxical questions. This article was well-received and generated many follow-up papers including some commentary by John Conway ([4]).

Finally, we highlight Landon's work applying graph theory to the semantic paradoxes, since many readers of this journal may be unfamiliar with the problem. Everyone will have heard of the Liar sentence: the sentence that says of itself that it is false. Notably this ancient paradox seems to rely on self-reference – a cyclical pattern of reference. But in the 1980s the philosopher Stephen Yablo demonstrated that the Liar paradox can be "unwound" into a infinite version that does not seem to involve referential circularity at all ([38], [39]). Briefly and informally, to get the idea, consider this infinite set of sentences:

Y_1: For each $i > 1$, Y_i is false.
Y_2: For each $i > 2$, Y_i is false.
Y_3: For each $i > 3$, Y_i is false.
$\vdots$

Is Y_1 true or false? In each case, we are led to a contradiction, just as with the Liar. (If all Y_i are false, then Y_1 is also true, giving a contradiction. So instead some Y_j must be true. Now, similar to before, Y_{j+1} is both false and true, again giving a contradiction.) But this paradox, known as Yablo's paradox, plunges us down an endless regress instead of trapping us in an endless loop. This raises the question: exactly what relations of reference afford the structure necessary for the semantic paradoxes? Landon helped to formulate this question as a characterization problem in infinite graph theory ([35]). Some partial results were obtained towards characterizing the "dangerous" graphs but the problem remains open ([6], [1]).

The papers in this special issue fall into three general categories. One set is papers related to Brooks' theorem and/or the Borodin-Kostochka conjecture:

- "Large cliques in graphs with high chromatic number" (Penny Haxell, Colter MacDonald)
- "Four proofs of the directed Brooks' Theorem" (Pierre Aboulker, Guillaume Aubian)

- "The list version of the Borodin-Kostochka conjecture for graphs with large maximum degree" (Ilkyoo Choi, H.A. Kierstead, Landon Rabern)
- "Yet another proof of Brooks' theorem" (Landon Rabern)

Another set is papers somehow related to generalizations of list coloring:

- "Generalized DP-colorings of graphs" (Alexandr V. Kostochka, Thomas Schweser, Michael Stiebitz)
- "A short proof that the list packing number of any graph is well defined" (Jeffrey A. Mudrock)
- "Kempe equivalent list edge-colorings of planar graphs" (Daniel W. Cranston)
- "Transformation invariance in the Combinatorial Nullstellensatz and nowhere-zero points of non-singular matrices" (Thomas Honold, Uwe Schauz)

And the final set is a varied assortment with no particular theme, reflecting Landon's diverse interests.

- "Coloring $(4K_1, C_4, C_6)$-free graphs" (Irena Penev)
- "Fermat's Last Theorem, Schur's Theorem (in Ramsey Theory), and the infinitude of the primes" (William Gasarch)
- "Saturation for the 3-Uniform Loose 3-Cycle" (Sean English, Alexandr Kostochka, Dara Zirlin)

Throughout his career, Landon expressed frustration at the slow pace of scientific progress, and his tireless efforts to make a difference were an inspiration to those around him. We believe that the papers in this volume reflect Landon's passion for mathematics and his remarkable talent, and we hope that they will serve as both a tribute to his legacy and an inspiration for future generations of mathematicians and computer scientists to pursue the elusive All Things Theorem. By picking up the tools and continuing Landon's work, we can honor his memory and contribute to the steady advancement of human knowledge.

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