Tree-ring semantics

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According to dendrochronology or tree-ring analysis, a science anticipated by da Vinci[1], the growth rings of a tree carry *information*. For example, as is well known, the number of growth rings in a tree cross-section represent the age of the tree. Here is a standard cross-section of a tree showing its center pith, a number of growth rings, and its outer bark:

Our aim here is to lay the groundwork for formal tree-ring analysis combining data from dendrochronology with formal techniques from semantics. We will present the basic syntax of, and basic compositional semantics of tree-ring structures.

First we define the formal tree-ring syntax. There are three basic symbols:

pith: $\bullet$

rings: $()$

bark: $\{\}$

The well-formed ring-structures of the language are divided into *ring-sentences* and *ring-terms*. They are provided by the following grammar, where each ϕ is a ring-sentence and each α is a ring-term:

$\alpha ::= (\bullet) \mid (\alpha)$
$\phi ::= \{\alpha\}$

Thus each ring-sentence is composed of bark encompassing a well-formed ring-term, where ring-terms are composed of any number of growth rings around a center pith. For example, a well-formed ring-sentence is the following: $\{((((\bullet))))\}$.

For the semantics let a model $\mathfrak{A} = \{\mathbb{N}, W, T, A\}$, where $\mathbb{N}$ is the natural numbers, W is a set of worlds and T is a set of times, and A is a set of individuals (or *trees*). Given this we provide the following lexical entries.

$[\![\bullet]\!] = 0$

$[\![()]\!] = \lambda n.n + 1$

$[\![\{\}]\!] = \lambda n.\{\langle w, t, a\rangle : a \text{ is } n \text{ years old in } w \text{ at } t\}$

[1] "Li circuli delli rami degli alberi segati mostrano il numero delli suoi anni, e quali furono più umidi o più secchi la maggiore o minore loro grossezza" (Leonardo da Vinci, *Trattato della Pittura*, 1817).

Composition proceeds via functional application.

> `Composition rule:` If χ is ring structure composed of immediate parts $\{\beta, \gamma\}$, and $\llbracket\gamma\rrbracket$ is in the domain of $\llbracket\beta\rrbracket$, then $\llbracket\chi\rrbracket = \llbracket\beta\rrbracket(\llbracket\gamma\rrbracket)$.

For example, consider the following sentence of the tree-ring language: $\{(((((((\bullet)))))))\}$. Given the semantics above we compute its truth conditions as follows:

$$
\begin{aligned}
\llbracket\{(((((((\bullet)))))))\}\rrbracket &= \llbracket\{\}\rrbracket(\llbracket(((((((\bullet)))))))\rrbracket)\\
&= \llbracket\{\}\rrbracket(\llbracket()\rrbracket(\llbracket((((((\bullet))))))\rrbracket))\\
&= \llbracket\{\}\rrbracket(1+(\llbracket((((((\bullet))))))\rrbracket))\\
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&= \llbracket\{\}\rrbracket(1+(1+(1+(1+(1+(1+(1+(0))))))))\\
&= \llbracket\{\}\rrbracket(7)\\
&= \{\langle w,t,a\rangle : a \text{ is 7 years old in } w \text{ at } t\}
\end{aligned}
$$

Thus we get the desired result that the sentence is true at a world centered on a tree and time just in case the tree is 7 years old in the world at that time.

Note that this initial investigation is quite limited in its ambitions. We have only addressed one aspect of tree-ring analysis. Our treatment only captures the core chronological information imparted by rings, the more complex information concerning environmental conditions or cross-dating via ring width, if it is even properly construed as semantic (instead of metasemantic), must await future work.

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