Subderivations

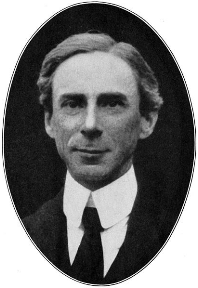
| University of Edinburgh | PHIL08004 |

$P \to Q \quad \therefore P \to ((Q \to R) \to R)$

1.	~~Show~~ $P \to ((Q \to R) \to R)$	
2.	P	ass cd
3.	~~Show~~ $(Q \to R) \to R$	
4.	$Q \to R$	ass cd
5.	$P \to Q$	pr1
6.	Q	2,5,mp
7.	R	4,6,mp
8.		7,cd
9.		3,cd

A clean-shaven male barber of a certain small town shaves all the inhabitants of the town who do not shave themselves, and never shaves any inhabitant who does shave himself.

Who shaves the barber?



[Bertrand Russell, *The Philosophy of Logical Atomism*]

$(T \rightarrow \sim P)$. $(\sim T \rightarrow \sim P)$. $\therefore \sim P$

1.	Show $\sim P$	
2.	$\sim\sim P$	ass id
3.	$\sim T$	pr1,2,mt
4.	$\sim\sim T$	pr2,2,mt
5.		3,4,id

$(T \rightarrow \sim P)$. $(\sim T \rightarrow \sim P)$. $\therefore \sim P$

1.	~~Show~~ $\sim P$	
2.	$\sim\sim P$	ass id
3.	$\sim T$	pr1,2,mt
4.	$\sim\sim T$	pr2,2,mt
5.		3,4,id

$P. \ {\sim}(Z \rightarrow P). \ \therefore {\sim}Z$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, ${\sim}Z$.

1.	Show ${\sim}Z$	
2.	Z	ass id
3.	Show $Z \rightarrow P$	
4.	Z	ass cd
5.	P	pr1
6.		5,cd

P. $\sim(Z \rightarrow P)$. $\therefore \sim Z$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\sim Z$.

1.	Show $\sim Z$	
2.	Z	ass id
3.	~~Sh~~ow $Z \rightarrow P$	
4.	Z	ass cd
5.	P	pr1
6.		5,cd
7.	$\sim(Z \rightarrow P)$	pr2

$P.\ \ {\sim}(Z \rightarrow P).\ \ \therefore {\sim}Z$

1.	~~Show~~ ${\sim}Z$	
2.	Z	ass id
3.	~~Show~~ $Z \rightarrow P$	
4.	Z	ass cd
5.	P	pr1
6.		5,cd
7.	${\sim}(Z \rightarrow P)$	pr2
8.		3,7,id

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1:62 [T16]  ∴ (¬P→¬Q)→(Q→P)
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1:91        ∴ ¬(P→¬P)→(¬P→P)
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1:98    pr1. ¬((U→T)→¬Q)
        pr2. ¬(P→(Q→R))→(Q→S)
        pr3. (¬(Q→R)→¬P)→((¬T→¬U)→S)
        ∴ S
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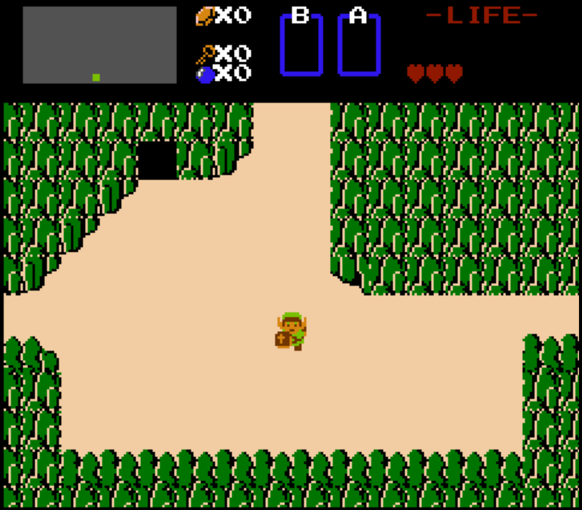
X0
X0
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