Quantifier negation rules

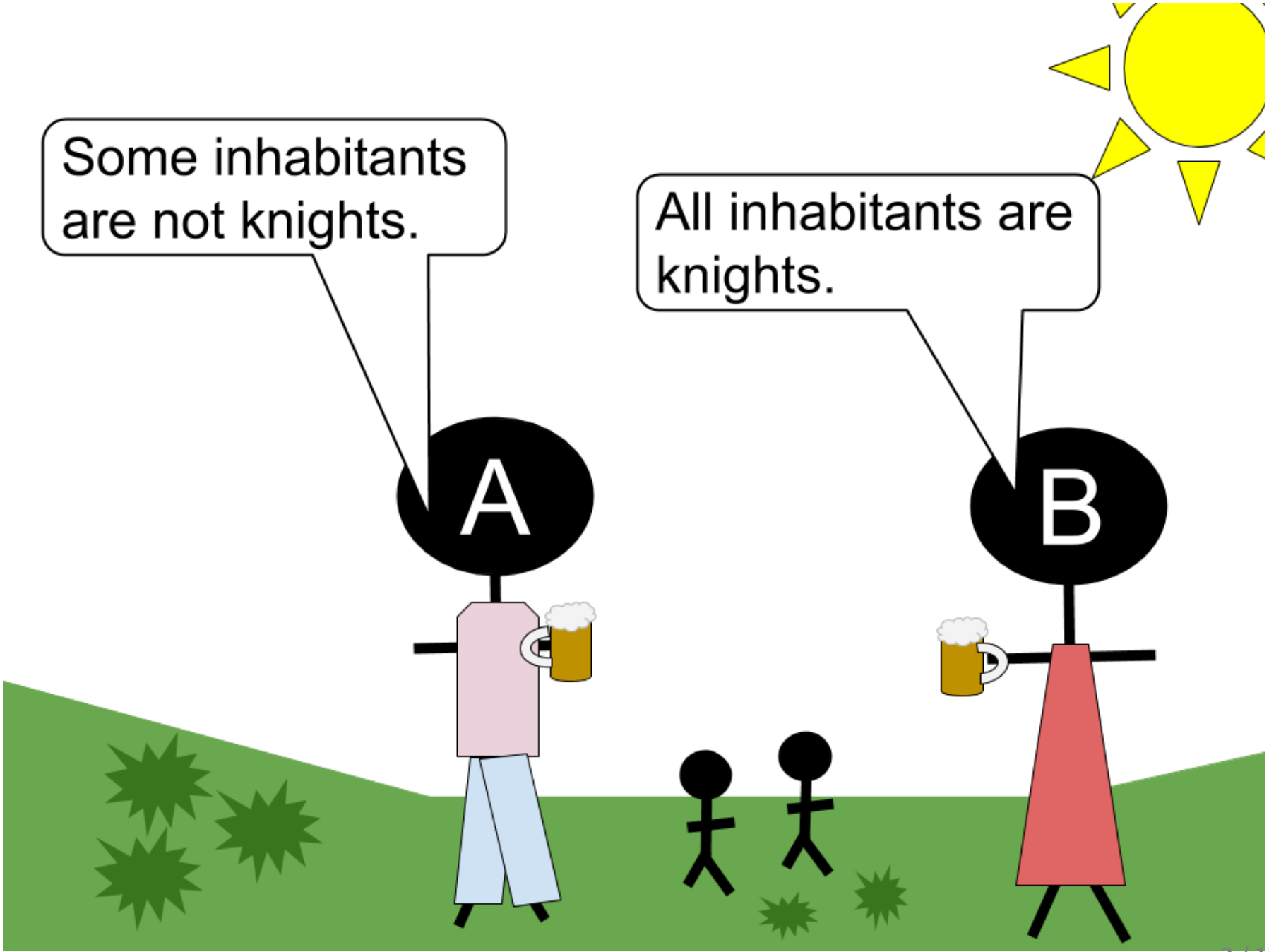
| University of Edinburgh | PHIL08004 |

Puzzle

The Island of Knights and Knaves: *There's an island in which certain inhabitants called "knights" always tell the truth, and others called "knaves" always lie. Every inhabitant of the island is either a knight or a knave.*

Puzzle. Two of the inhabitants—A and B—were standing together in a garden having a beer. A says, "Some inhabitants of this island are not knights". B says "All inhabitants of this island are knights".

The question is, what are A and B?


Some inhabitants are not knights.
All inhabitants are knights.
A
B

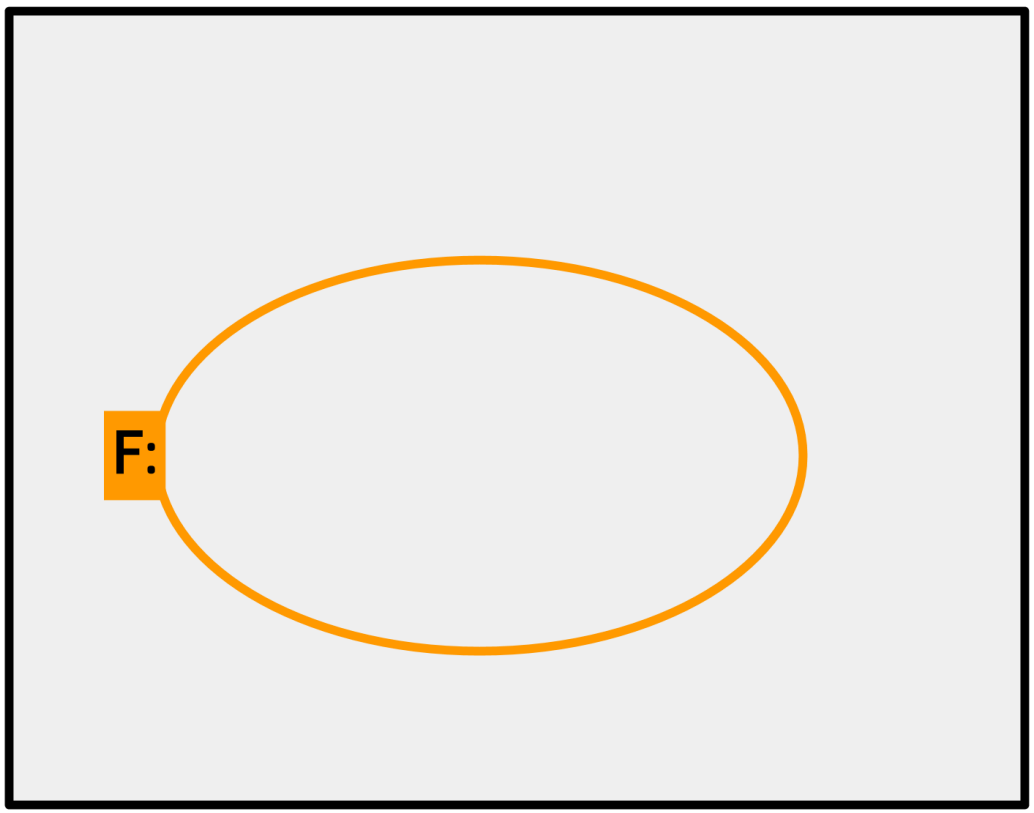
$$\neg\exists x \neg Hx$$

$$\forall x Hx$$

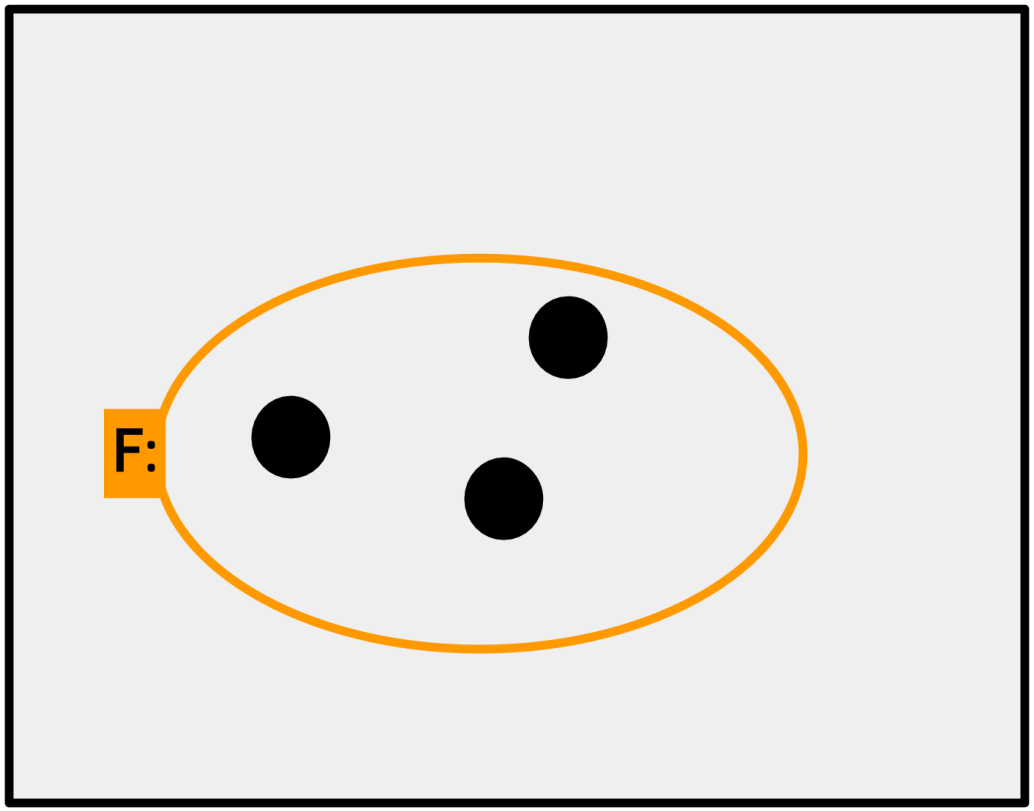
Apply inference rules to get:

$$Ha$$

$\neg\exists x \neg Fx$

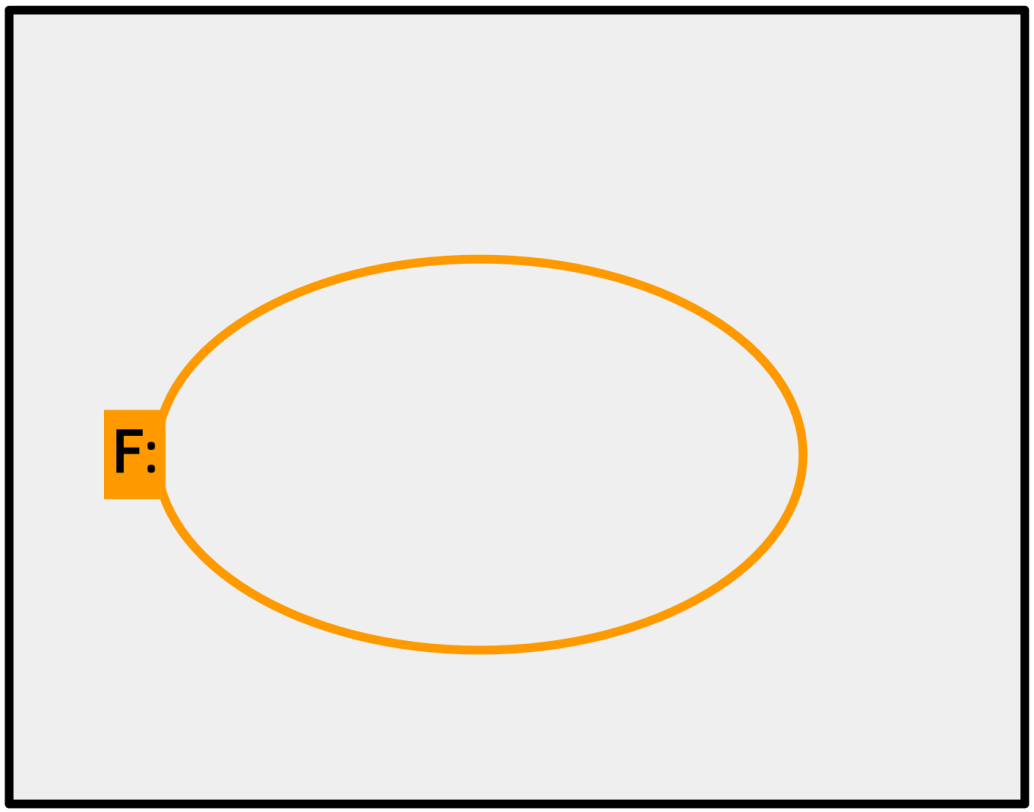


$\neg\exists x\neg Fx$

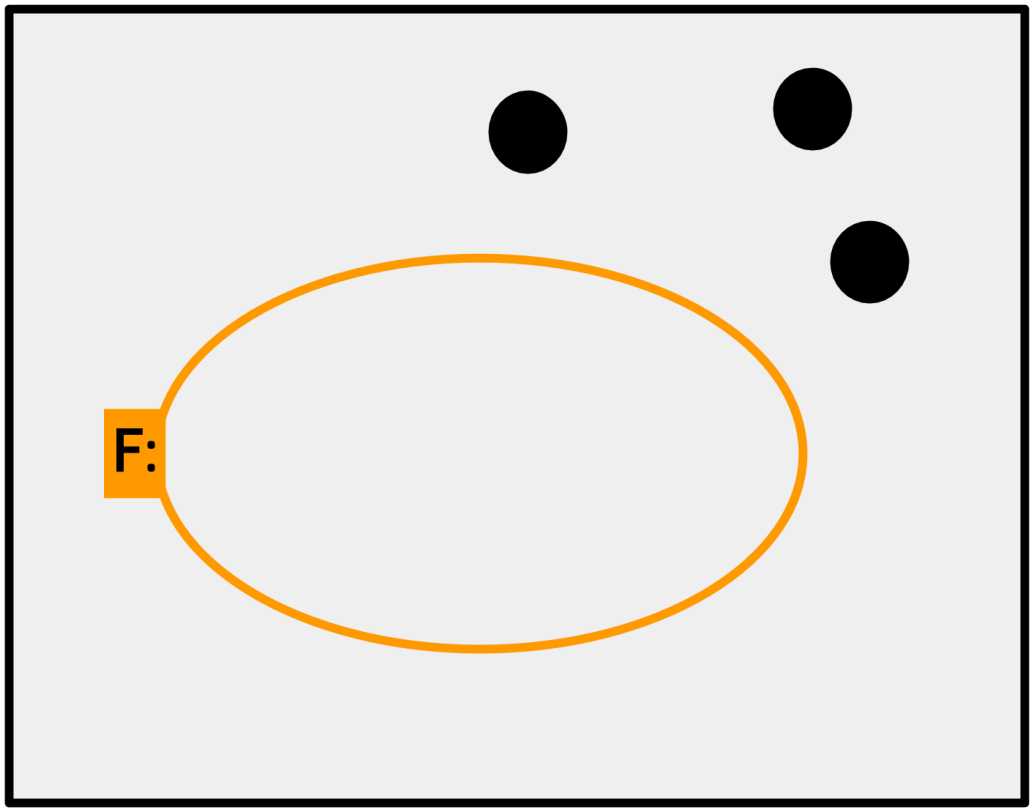


$\forall xFx$

$\neg\exists x Fx$

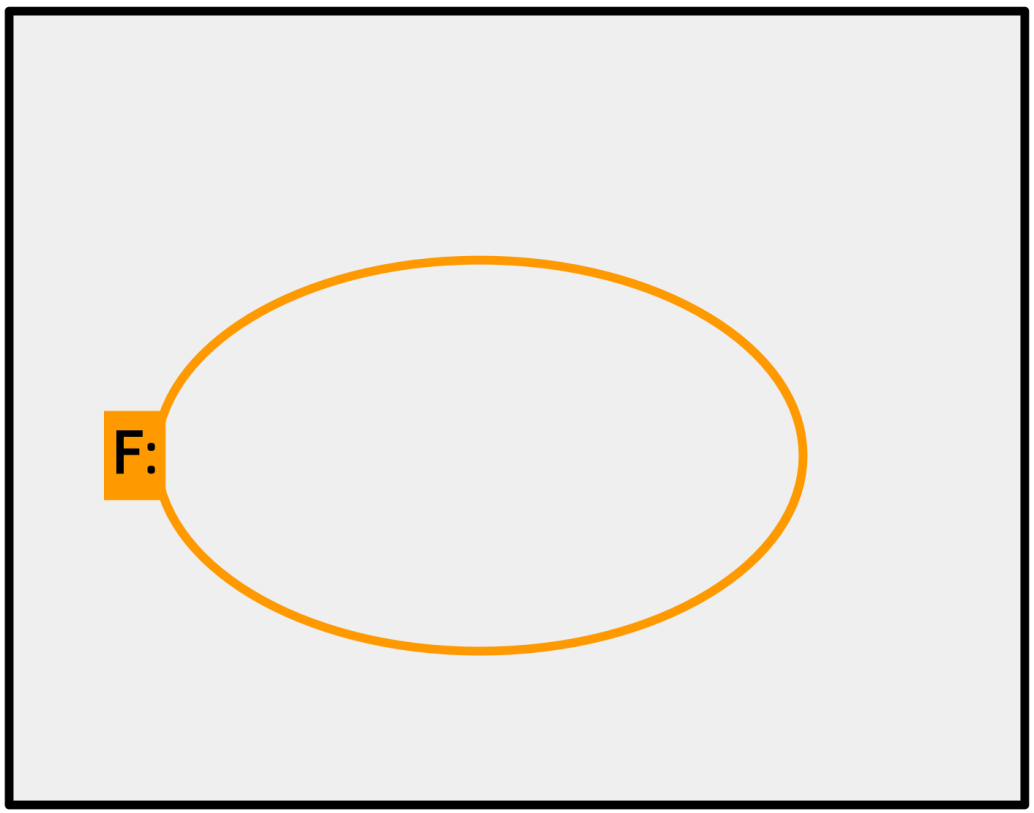


$\neg\exists x Fx$

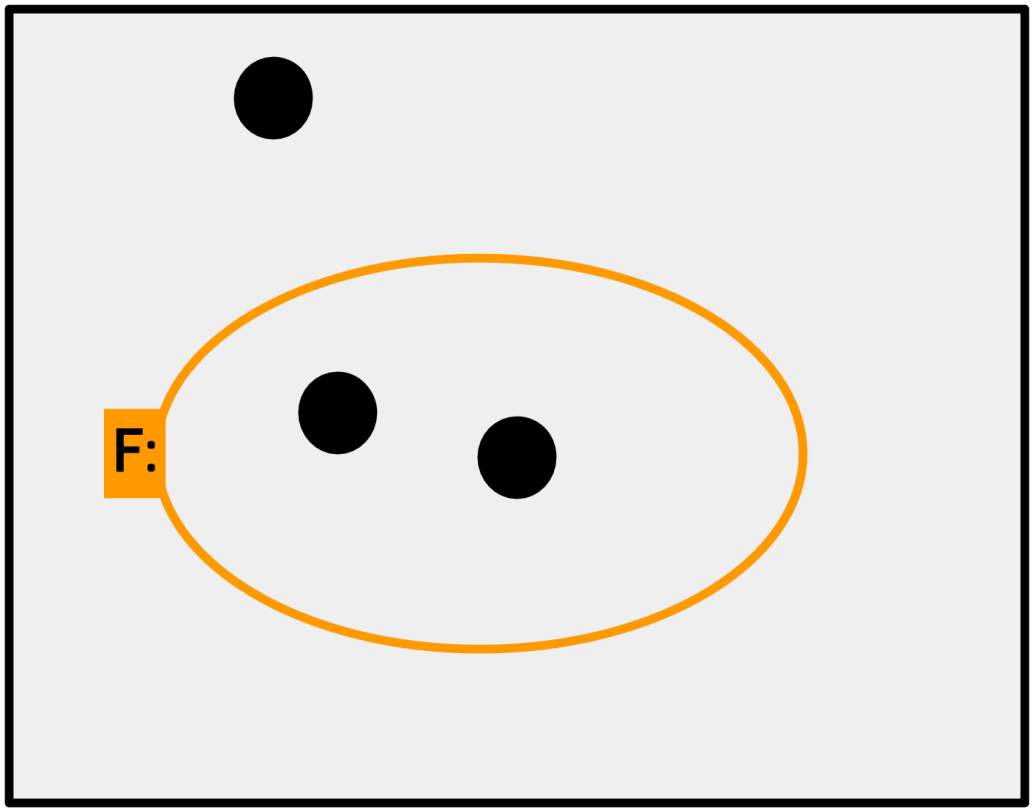


$\forall x \neg Fx$

$\neg\forall xFx$

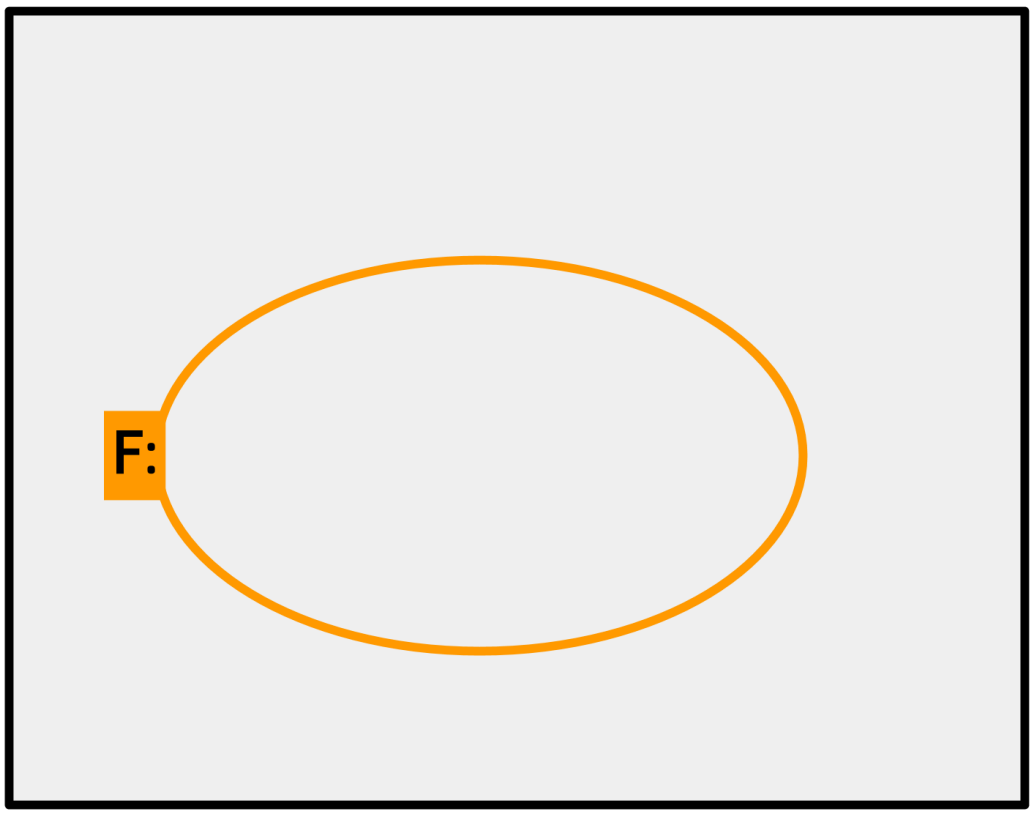


$\neg\forall xFx$

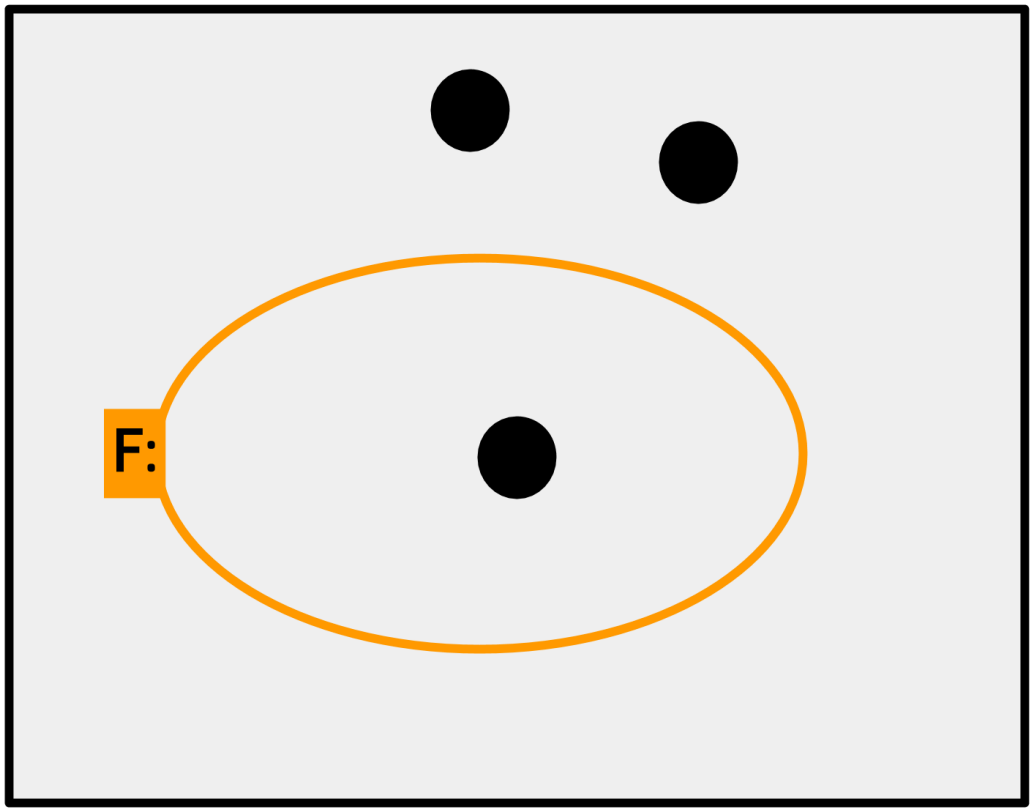


$\exists x\neg Fx$

$\neg\forall x\neg Fx$



$\neg\forall x\neg Fx$



$\exists xFx$

QUANTIFIER NEGATION RULES (T203-T206)

$$\frac{\neg\forall x\phi}{\exists x\neg\phi}\ \textbf{qn} \qquad \frac{\forall x\phi}{\neg\exists x\neg\phi}\ \textbf{qn}$$

$$\frac{\neg\forall x\neg\phi}{\exists x\phi}\ \textbf{qn} \qquad \frac{\forall x\neg\phi}{\neg\exists x\phi}\ \textbf{qn}$$

$$\frac{\neg\exists x\phi}{\forall x\neg\phi}\ \textbf{qn} \qquad \frac{\exists x\phi}{\neg\forall x\neg\phi}\ \textbf{qn}$$

$$\frac{\neg\exists x\neg\phi}{\forall x\phi}\ \textbf{qn} \qquad \frac{\exists x\neg\phi}{\neg\forall x\phi}\ \textbf{qn}$$

$$\neg \forall x Fx \leftrightarrow \exists x \neg Fx$$

1	**Show** $\neg\forall x Fx \leftrightarrow \exists x \neg Fx$	
2	**Show** $\neg\forall x Fx \rightarrow \exists x \neg Fx$	
3	$\neg\forall x Fx$	ass cd
4	**Show** $\exists x \neg Fx$	
5	$\neg\exists x \neg Fx$	ass id
6	**Show** $\forall x Fx$	
7	**Show** Fx	
8	$\neg Fx$	ass id
9	$\exists x \neg Fx$	8 , eg , x , x
10		5 , 9 , id
11		7 , ud
12		3 , 6 , id
13		4 , cd
14	**Show** $\exists x \neg Fx \rightarrow \neg\forall x Fx$	
15	$\exists x \neg Fx$	ass cd
16	**Show** $\neg\forall x Fx$	
17	$\forall x Fx$	ass id
18	$\neg Fz$	15 , ei , z
19	Fz	17 , ui , z
20		18 , 19 , id
21		16 , cd
22	$\neg\forall x Fx \leftrightarrow \exists x \neg Fx$	2 , 14 , cb
23		22 , dd