$\mathcal{L}_2$ inference rules

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Conjunction rules:

Rule s (simplification)

$$\begin{array}{ll} & \Box \wedge \bigcirc \\ \therefore & \Box \end{array} \quad \text{or} \quad \begin{array}{ll} & \Box \wedge \bigcirc \\ \therefore & \bigcirc \end{array}$$

Rule adj (adjunction)

$$\begin{array}{ll} & \Box \\ & \bigcirc \\ \therefore & \Box \wedge \bigcirc \end{array}$$

Disjunction rules:

Rule add (addition)

$$\begin{array}{ll} & \Box \\ \therefore & \Box \vee \bigcirc \end{array} \quad \text{or} \quad \begin{array}{ll} & \Box \\ \therefore & \bigcirc \vee \Box \end{array}$$

Rule mtp (modus tollendo ponens)

$$\begin{array}{ll} & \Box \vee \bigcirc \\ & {\sim}\bigcirc \\ \therefore & \Box \end{array} \quad \text{or} \quad \begin{array}{ll} & \Box \vee \bigcirc \\ & {\sim}\Box \\ \therefore & \bigcirc \end{array}$$

Biconditional rules:

Rule bc (biconditional-to-conditional)

$$\begin{array}{ll} & \Box \leftrightarrow \bigcirc \\ \therefore & \Box \rightarrow \bigcirc \end{array} \quad \text{or} \quad \begin{array}{ll} & \Box \leftrightarrow \bigcirc \\ \therefore & \bigcirc \rightarrow \Box \end{array}$$

Rule cb (conditionals-to-biconditional)

$$\begin{array}{ll} & \Box \rightarrow \bigcirc \\ & \bigcirc \rightarrow \Box \\ \therefore & \Box \leftrightarrow \bigcirc \end{array}$$

Alfred reads books ∴ Alfred reads books **or** God is an alien.

`rule add`

<u>Addition</u>: from any sentence you may infer its disjunction with any other sentence.

add:

$\Box$

$(\Box \vee \bigcirc)$
$(\bigcirc \vee \Box)$

Either Alfred does yoga **or** Elle does yoga. But Alfred doesn't do yoga $\therefore$ Elle does yoga.

`rule mtp`

> Modus tollendo ponens: from a disjunction and the negation of one of its disjuncts you may infer the other disjunct.

mtp:

$(\square \lor \bigcirc)$

$\sim\bigcirc$

$\square$

Alfred reads books and Alfred does yoga. $\therefore$ Alfred does yoga.

`rule s`

Simplification: if you have a conjunction, you may infer either conjunct.

s:

$(\Box \wedge \bigcirc)$

$\Box$

$\bigcirc$

God is made of spaghetti. God owns a velociraptor.
$\therefore$ God is made of spaghetti and owns a velociraptor.

`rule adj`

> Adjunction: if you have any two sentences, you may infer their conjunction, in either order.

adj:

$\square$

$\bigcirc$

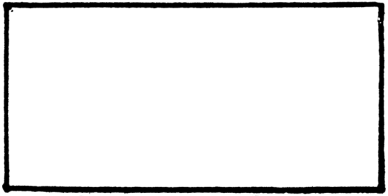
$(\square \wedge \bigcirc)$
$(\bigcirc \wedge \square)$

Biconditional

- "James is angry if and only if Ann is happy"
- 'if and only if' is a connective used to combine two propositions
    - A biconditional says that the truth of either proposition requires the truth of the other, i.e., either both propositions are true, or both are false
- P **iff** Q
    - $(P \rightarrow Q)$ and $(Q \rightarrow P)$
- $(P \leftrightarrow Q)$

iff

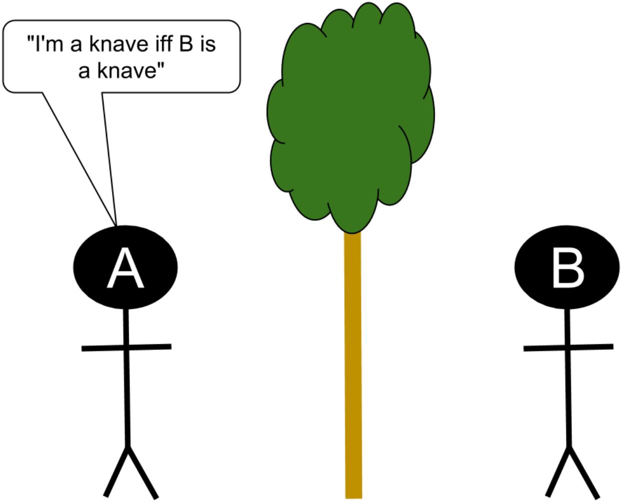
- x is a rectangle iff:
  - x is a polygon — necessary but not sufficient
  - x has four sides — necessary but not sufficient
  - x is a quadrilateral with right angles — necessary and sufficient
  - x is a square — sufficient but not necessary



Rectangle.

Puzzle. Two of the inhabitants—A and B—are standing together under a tree. A makes the following statement: "I am a knave iff B is a knave."

What is B?


"I'm a knave iff B is a knave"
A
B

rule bc

<u>Biconditional-to-conditional</u>: from a biconditional you may infer either of the corresponding conditionals.

bc:

$$(\Box \leftrightarrow \bigcirc)$$

$$(\Box \rightarrow \bigcirc)$$
$$(\bigcirc \rightarrow \Box)$$

`rule cb`

Conditionals-to-biconditional: from two conditionals where the antecedent of one is the consequent of the other, and vice versa, you may infer a biconditional containing the parts of the conditionals.

cb:

$(\square \rightarrow \bigcirc)$
$(\bigcirc \rightarrow \square)$

$(\square \leftrightarrow \bigcirc)$

Practice exercises

2:1 $(P \vee R)$. $(\neg R \wedge T)$. $(P \vee (Q \wedge T)) \to S \therefore S$

2:3 $(P \leftrightarrow \neg Q) \to R \therefore (\neg R \wedge P) \to Q$

2:4 $\neg(\neg R \to (Q \wedge T))$. $\neg S \vee R \therefore \neg S$