

Direct, Conditional, and Indirect Derivations

| University of Edinburgh | PHIL08004 |

$P \rightarrow \sim Q. Z \rightarrow X. \sim Z \rightarrow Q. \sim X \therefore \sim P$

1.	Show $\sim P$	
2.	$Z \rightarrow X$	pr2
3.	$\sim X$	pr4
4.	$\sim Z$	2,3,mt
5.	$\sim Z \rightarrow Q$	pr3
6.	Q	4,5,mp
7.	$\sim \sim Q$	6,dn
8.	$P \rightarrow \sim Q$	pr1
9.	$\sim P$	7,8,mt
10.		9,dd



$P. (Q \rightarrow \sim P). (R \rightarrow Q). \therefore \sim R$

- | | | |
|----|------------------------|--------|
| 1. | Show $\sim R$ | |
| 2. | P | pr1 |
| 3. | $\sim\sim P$ | 2,dn |
| 4. | $Q \rightarrow \sim P$ | pr2 |
| 5. | $\sim Q$ | 3,4,mt |
| 6. | $R \rightarrow Q$ | pr3 |
| 7. | $\sim R$ | 5,6,mt |
| 8. | | 7,dd |

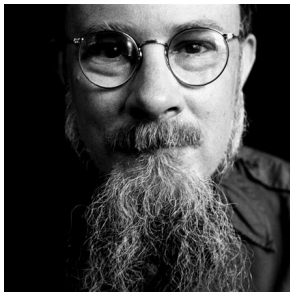
$P. (Q \rightarrow \sim P). (R \rightarrow Q). \therefore \sim R$

1. Show $\sim R$

2.	P	pr1
3.	$\sim\sim P$	2,dn
4.	$Q \rightarrow \sim P$	pr2
5.	$\sim Q$	3,4,mt
6.	$R \rightarrow Q$	pr3
7.	$\sim R$	5,6,mt
8.		7,dd

Direct derivation (dd)

When a line (which is not a show line) is introduced whose sentence is the same as the sentence on the (closest previous uncanceled) show line, one may, as the next step, write “dd” following the justification for that line, draw a line through the word “Show”, and draw a box around all the lines below the show line, including the current line.



David Lewis

“Standard versions of the argument from evil concern the evils God fails to prevent: the pain and suffering of human beings and non-human animals, and the sins people commit.”

“But we might start instead from the evils God himself perpetrates. There are plenty of these, and, in duration and intensity, they dwarf the kinds of suffering and sin to which the standard versions allude.”



Fire and brimstone

- ▶ “Depart from me, you who are cursed, into the eternal fire prepared for the devil and his angels” (Matthew 25:41)
- ▶ “...be thrown into hell, where ‘the worms that eat them do not die, and the fire is not quenched’. Everyone will be salted with fire. (Mark 9:48-49)
- ▶ “He will punish those who do not know God and do not obey the gospel of our Lord Jesus. They will be punished with everlasting destruction” (2 Thessalonians 1:8-9)



Divine evil

- ▶ (1) If God condemns the insubordinate to Hell, he inflicts punishment that is vastly disproportionate to the offence.
- ▶ (2) If God inflicts punishment that is vastly disproportionate to the offence, then God perpetrates unjust evil.
- ▶ (3) Thus, if God condemns the insubordinate to Hell, then God perpetrates unjust evil.

P: God condemns the insubordinate to Hell, Q: God inflicts punishment that is vastly disproportionate to the offence, V: God perpetrates unjust evil.

$$(P \rightarrow Q). (Q \rightarrow V) \therefore (P \rightarrow V)$$

$$(P \rightarrow Q). (Q \rightarrow V) \therefore (P \rightarrow V)$$

- | | | |
|----|--------------------------|--------|
| 1. | Show $(P \rightarrow V)$ | |
| 2. | P | ass cd |
| 3. | $(P \rightarrow Q)$ | pr1 |
| 4. | Q | 2,3,mp |
| 5. | $(Q \rightarrow V)$ | pr2 |
| 6. | V | 4,5,mp |
| 7. | | 6,cd |

Assume P is true. Since if P , then Q , it follows that Q . And since if Q , then V , it follows that V . So summing up, if P , then V .

“Conditional Derivation”

$$(P \rightarrow Q). (Q \rightarrow V) \therefore (P \rightarrow V)$$

1. Show $(P \rightarrow V)$

2.

P

ass cd

3.

$(P \rightarrow Q)$

pr1

4.

Q

2,3,mp

5.

$(Q \rightarrow V)$

pr2

6.

V

4,5,mp

7.

6,cd

“Conditional Derivation”

Conditional derivation (cd)

Assumption for conditional derivation: When a show line with a conditional sentence is introduced, as the next step one may introduce a following line with the antecedent of the conditional on it; the justification is “ass cd”.

Conditional derivation: When a line (which is not a show line) is introduced whose sentence is the same as the consequent of the conditional sentence on the (closest previous uncanceled) show line, one may, as the next step, write “cd” at the end of that line, draw a line through the word “Show”, and draw a box around all the lines below the show line, including the current line.

$$P \therefore ((P \rightarrow Q) \rightarrow Q)$$

1. Show $((P \rightarrow Q) \rightarrow Q)$
2. $(P \rightarrow Q)$ ass cd
3. P pr1
4. Q 2,3,mp
5. 4,cd

**ASSUMED THE ANTECEDENT, DERIVED THE
CONSEQUENT**



BOX AND CANCEL

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$P \therefore ((P \rightarrow Q) \rightarrow Q)$

1. Show $((P \rightarrow Q) \rightarrow Q)$

2. $(P \rightarrow Q)$

ass cd

3. P

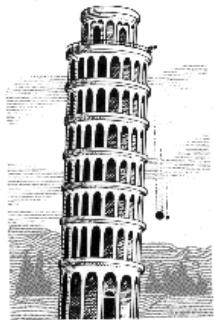
pr1

4. Q

2,3,mp

5.

4,cd



Heavier objects don't fall faster than lighter ones.

- ▶ Heavier objects don't fall faster than lighter ones.



Galileo's Argument

Salviati: If we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?

Simplicio: You are unquestionably right.

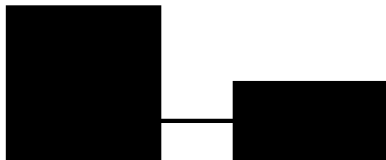
Salviati: But if this is true, and if a large stone moves with a speed of, say, eight, while a smaller stone moves with a speed of four, then when they are united, the system will move with a speed of less than eight. Yet the two stones tied together make a stone larger than that which before moved with a speed of eight: hence the heavier body now moves with less speed than the lighter, an effect which is contrary to your supposition. Thus you see how, from the assumption that the heavier body moves faster than the lighter one, I can infer that the heavier body moves more slowly. . .

And so, Simplicio, we must conclude therefore that large and small bodies move with the same speed...

[Galileo Galilei, *Discorsi e Dimostrazioni Matematiche* (1628)]

Galileo's Argument

- ▶ Assume that heavier objects fall faster than lighter objects.



- ▶ When united, the larger one will be slowed down by the smaller one, so the two-stone system will move **slower**
- ▶ But when united the stones compose an object that is heavier than either of the original stones, so the two-stone system will move **faster**

Galileo's Argument

- ▶ (1) If heavier objects fall faster than lighter objects, then stone A falls faster than stone B.
- ▶ (2) If stone A falls faster than stone B, then the system of stones A+B does not fall faster than stone A alone. (The slow moving B will slow down A.)
- ▶ (3) If heavier objects fall faster than lighter objects, then the system of stones A+B falls faster than stone A alone. (The system A+B is heavier than A alone.)
- ▶ (4) Thus, it is not the case that heavier objects fall faster than lighter objects

P: Heavier objects fall faster than lighter objects, Q: Stone A falls faster than stone B, S: The system of stones A+B falls faster than stone A alone.

$$(P \rightarrow Q). (Q \rightarrow \sim S). (P \rightarrow S) \therefore \sim P$$

$$(P \rightarrow Q). (Q \rightarrow \sim S). (P \rightarrow S) \therefore \sim P$$

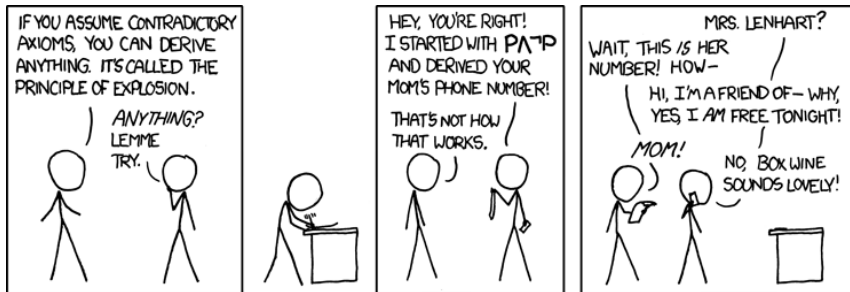
- | | | |
|----|--------------------------|--------|
| 1. | Show $\sim P$ | |
| 2. | P | ass id |
| 3. | $(P \rightarrow Q)$ | pr1 |
| 4. | Q | 2,3,mp |
| 5. | $(Q \rightarrow \sim S)$ | pr2 |
| 6. | $\sim S$ | 4,5,mp |
| 7. | $(P \rightarrow S)$ | pr3 |
| 8. | S | 2,7,mp |
| 9. | | 6,8,id |

Assume the opposite, i.e. assume P is true. Then since if P then Q , it follows that Q . And since if Q then $\sim S$, it follows that $\sim S$.

$$(P \rightarrow Q). (Q \rightarrow \sim S). (P \rightarrow S) \therefore \sim P$$

1. Show $\sim P$

2.	P	ass id
3.	$(P \rightarrow Q)$	pr1
4.	Q	2,3,mp
5.	$(Q \rightarrow \sim S)$	pr2
6.	$\sim S$	4,5,mp
7.	$(P \rightarrow S)$	pr3
8.	S	2,7,mp
9.		6,8,id



Ex falso quodlibet

The principle of explosion: “from a contradiction, anything follows”

Indirect derivation (id)

Assumption for indirect derivation: When a show line is introduced, as the next step one may introduce a following line with the [un]negation of the sentence on the show line; the justification is “ass id”.

Indirect derivation: When a sentence is introduced on a line which is not a show line, if there is a previous available line containing the [un]negation of that sentence, and if there is no uncanceled show line between the two sentences, as the next step you may write the line number of the first sentence followed by “id” at the end of the line with the second sentence. Then you cancel the closest previous “show”, and box all sentences below that show line, including the current line.