Derivations

| University of Edinburgh | PHIL08004 |

pr1	¬¬(P → Q)	pr
pr2	P	pr
1	**Show** Q	
2	¬Q	ass id
3	P → Q ⊕	pr1 , dn
4	P	pr2 , r
5	Q	3 , 4 , mp

Arguments

An **argument** is a sequence of sentences, consisting of premises and a conclusion, where the conclusion is what is trying to be established, and the premises, taken together, are alleged to support the conclusion.

- **Argument:** A sequence of sentences consisting of at least 0 premises and exactly 1 conclusion.
- $\square_1. \ldots \square_n \therefore \bigcirc$
- $(P \rightarrow {\sim}Q).\ Q \therefore\ {\sim}P$

Demonstrating validity

An argument is **valid** just in case if its premises are true, then its conclusion must also be true.

- When an argument is valid its conclusion "follows from" its premises.
- But what follows from what isn't always obvious.
- Complex reasoning often consists of the stringing together of many simple inferences.
- A demonstration of *how* the conclusion follows is often helpful.
- Such a demonstration is a "**proof**" or "**derivation**".

Derivations

Intuitively, a derivation is a sequence of intermediate steps—each of which constitutes a valid argument—that leads from the premises of an argument to the conclusion of the argument.

In a formal system a **derivation** is an explicit line by line demonstration of how to produce the conclusion starting from the premises in accordance with the axioms and inference rules of the system.

Natural deduction (Gentzen)

"My starting point was this: The formalization of logical deduction, especially as it has been developed by Frege, Russell, and Hilbert, is rather far removed from the forms of deduction used in practice in mathematical proofs. In contrast, I intended first to set up a formal system which comes as close as possible to actual reasoning. The result was a **calculus of natural deduction**. . ." [Gerhard Gentzen (1934) "Untersuchungen über das logische Schließen"]

Natural deduction (Jaśkowski)

In 1926 Łukasiewicz noted in his seminars that mathematicians do not actually construct their proofs by means of an axiomatic theory (in the style of Frege), but instead reason in more natural ways, e.g making assumptions and reasoning form there. His student Jaśkowski, developed the desired natural system in "On the Rules of Suppositions in Formal Logic" (1934).

Inference rules

The inference rules model our natural forms of valid reasoning. In our formal system the rules specify what symbolic transformations are allowed.

mp

$\square \to \bigcirc$

$\square$

$\bigcirc$

mt

$\square \to \bigcirc$

$\sim\bigcirc$

$\sim\square$

dn

$\square$

$\sim\sim\square$

$\square$

r

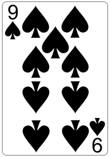
$\square$

$\square$

If your card has a red back, your card is an Ace. But your card isn't an Ace. Thus, your card does not have a red back.

- R: Your card has a red back
- Z: Your card is an Ace.

$(R \to Z)$. ~Z $\therefore$ ~R



$(R \rightarrow Z).\ \ {\sim}Z \quad \therefore {\sim}R$

- The reasoning from premises to conclusion via the inference rules can be systematised and explained.
- A derivation carries out the task of showing that a sentence follows from certain premises.

Show line

Derivations begin with a special line stating the task; that is, **stating what is to be shown**.

1. Show ~*R*

A "show" line may be introduced at any time, and it does not need a justification, because it only states what it is we intend to derive. All other lines need justifications.

Justified lines: premises and rules

> The other lines of a derivation consists of a sentence followed by a justification.

1.	Show ~R	
2.	$(R \rightarrow Z)$	pr1
3.	~Z	pr2
4.	~R	2,3,mt

Every line consists of a line number followed by a sentence followed by a justification. The sentence on each line either (i) occurs as a premise, and the line is justified by writing "pr", or (ii) follows from previous lines by a rule, and the line is justified by writing the number(s) of the line(s) from which it follows, along with a short name of the rule.

Box and cancel

> A derivation ends with **boxing and cancelling**.

1.	Show ~R	
2.	$(R \rightarrow Z)$	pr1
3.	~Z	pr2
4.	~R	2,3,mt
5.		4, dd

The cancellation of the show line indicates that the task was successfully completed. We indicate what kind of derivation it is ("dd", "id", "cd") after the final line. The steps used in that part of the derivation are *boxed* off.

Box and cancel

A derivation ends with **boxing and cancelling**.

1.	Show ~R	
2.	$(R \rightarrow Z)$	pr1
3.	~Z	pr2
4.	~R	2,3,mt
5.		4, dd

The cancellation of the show line indicates that the task was successfully completed. We indicate what kind of derivation it is ("dd", "id", "cd") after the final line. The steps used in that part of the derivation are *boxed* off.

Box and cancel

A derivation ends with **boxing and cancelling**.

1.	~~Show~~ $\sim R$	
2.	$(R \rightarrow Z)$	pr1
3.	$\sim Z$	pr2
4.	$\sim R$	2,3,mt
5.		4, dd

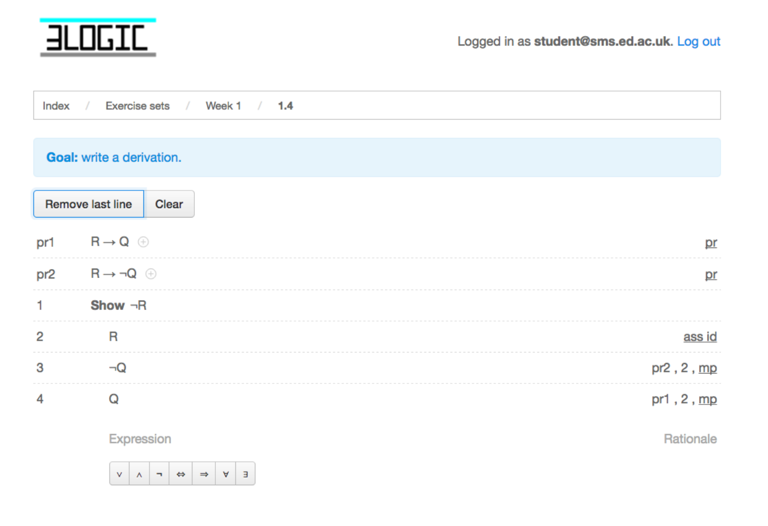
The cancellation of the show line indicates that the task was successfully completed. We indicate what kind of derivation it is (“dd”, “id”, “cd”) after the final line. The steps used in that part of the derivation are *boxed* off.

Completed derivation

$(R \to Z).\ \ {\sim}Z \quad \therefore {\sim}R$

1.	~~Show~~ ${\sim}R$	
2.	$(R \to Z)$	pr1
3.	${\sim}Z$	pr2
4.	${\sim}R$	2,3,mt
5.		4,dd

∃LOGIC: Derivations



$(P \to {\sim}Q).\ (Z \to X).\ ({\sim}Z \to Q).\ {\sim}X \ \therefore\ {\sim}P$

1.	Show ${\sim}P$	
2.	$Z \to X$	pr2
3.	${\sim}X$	pr4
4.	${\sim}Z$	2,3,mt
5.	${\sim}Z \to Q$	pr3
6.	Q	4,5,mp
7.	${\sim}{\sim}Q$	6,dn
8.	$P \to {\sim}Q$	pr1
9.	${\sim}P$	7,8,mt

$P \to {\sim}Q.\ \ Z \to X.\ \ {\sim}Z \to Q.\ \ {\sim}X\ \ \therefore {\sim}P$ Correct

1.	~~Show~~ ${\sim}P$	
2.	$Z \to X$	pr2
3.	${\sim}X$	pr4
4.	${\sim}Z$	2,3,mt
5.	${\sim}Z \to Q$	pr3
6.	Q	4,5,mp
7.	${\sim}{\sim}Q$	6,dn
8.	$P \to {\sim}Q$	pr1
9.	${\sim}P$	7,8,mt
10.		9,dd

Construct direct derivations to validate each of the following arguments:

P
$Q \rightarrow \sim P$
$R \rightarrow Q$
$\therefore \sim R$

$W \rightarrow \sim(V \rightarrow \sim Y)$
$X \rightarrow (V \rightarrow \sim Y)$
$V \rightarrow Y$
$(V \rightarrow Y) \rightarrow X$
$\therefore \sim W$

$(W \rightarrow Z) \rightarrow (Z \rightarrow W)$
$(Z \rightarrow W) \rightarrow \sim X$
$P \rightarrow X$
$\sim\sim P$
$\therefore \sim(W \rightarrow Z)$

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1:3

¬ P. (Q → P) ∴ ¬ Q

1:4

¬ ¬ (P → Q). P ∴ Q

1:5

P. (R → ¬ Q). (P → Q) ∴ ¬ R

1:9

R → Q. (R → ¬ Q) ∴ ¬ R

1:10

P → (Q → ¬ R). Q ∴ P → ¬ R