

The language of 'if' and 'not'

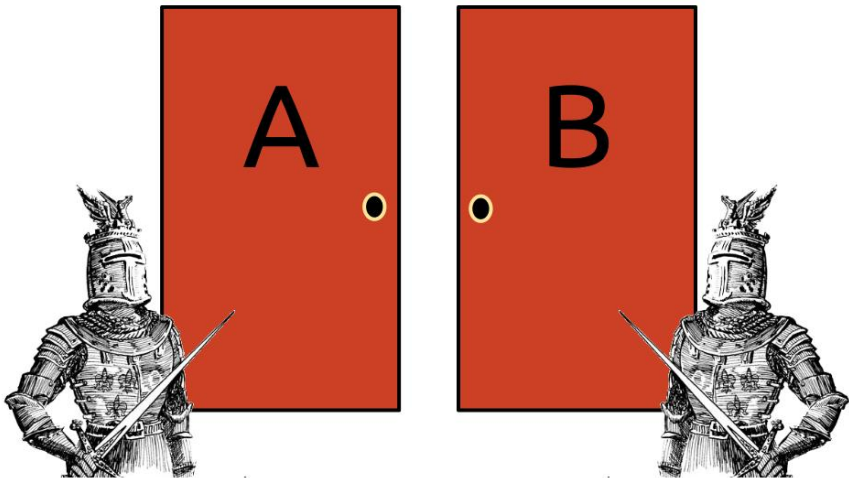
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$$P \rightarrow Q. \sim Q \quad \therefore \quad \sim P$$



Puzzle 4: The Guards

- ▶ Two doors: A and B
- ▶ One leads to Heaven; one leads to Hell
- ▶ Two guards: guard of A and guard of B
- ▶ One lies and one tells the truth
- ▶ One yes/no question to ask
- ▶ What do you ask?



Solution

- ▶ Ask guard A the following:
 - ▶ **If I asked guard B whether his door leads to heaven would he say 'yes'?**

- ▶ **YES**
 - ▶ A is telling truth: B would say 'yes' and B is lying, so its door A.
 - ▶ A is lying: B would say 'no' and B is telling truth, so door A.

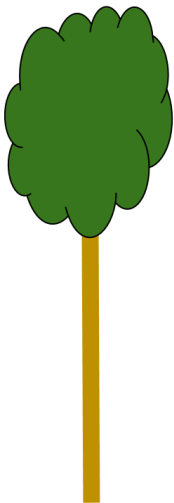
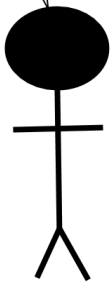
- ▶ **NO**
 - ▶ A is telling truth: B would say 'no' and B is lying, so its door B.
 - ▶ A is lying: B would say 'yes' and B is telling truth, so door B.

Not Not Knight

Puzzle 5. You come across an inhabitant who makes the following statement “It is not the case that I am not a knight.”

Can you determine whether the inhabitant is a knight or a knave?

"It is not the case that
I'm not a knight"

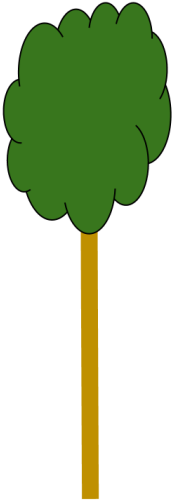


The King

Puzzle 6. Assume that there is a King of the Island of Knights and Knaves. Who is the King? You come across two inhabitants A and B and A makes the following statement: “If I’m a knight, then B is the King”.

Is B the King?

"If I'm a knight,
then B is the King"



A: If I'm a knight, then B is the King

- ▶ Assume A is a knave
- ▶ So what A said is false
- ▶ So: A is a knight but B is not the King
 - ▶ Contradiction!
- ▶ So A is a knight
- ▶ And, so **B is the King**

\mathcal{L}_1 : The language of 'if' and 'not'

▶ BASIC SYMBOLS:

- ▶ Sentence letters: P, \dots, Z
 - ▶ (including numerical subscripts, e.g. P_4, Q_{23} , etc.)
- ▶ Connectives: \sim, \rightarrow
- ▶ Punctuation: $), ($

▶ FORMATION RULES:

- ▶ Any sentence letter is a sentence
- ▶ If \square is a sentence, then $\sim\square$ is a sentence
- ▶ If \square and \circ are a sentences, then $(\square \rightarrow \circ)$ is a sentence
- ▶ Nothing else is a sentence of \mathcal{L}_1 unless it can be constructed by means of these rules.

Terminology

- ▶ **Atomic sentence:** A sentence containing no connectives at all, e.g. Q .
- ▶ **Molecular sentence:** A sentence containing at least one connective. e.g. $\sim Q$.
- ▶ **Negation:** $\sim \square$ is the negation of \square
 - ▶ \square is the unnegation of $\sim \square$
- ▶ **Conditional:** A sentence of the form $(\square \rightarrow \circ)$ is a conditional
 - ▶ In a conditional $(\square \rightarrow \circ)$, \square is called the antecedent and \circ is called the consequent.

Well-formed?

- ▶ $P \sim Q (\rightarrow (R))$
- ▶ R
- ▶ A
- ▶ $(U \rightarrow Z)$
- ▶ $\sim R$
- ▶ $\sim(P \rightarrow R)$
- ▶ $P \rightarrow Q$ (but later we will adopt an informal convention)
- ▶ $\sim(P)$

Well-formed?

▶ $\sim(P \rightarrow Q)$

▶ $(\sim P \rightarrow Q)$

▶ $(P \rightarrow \sim Q)$

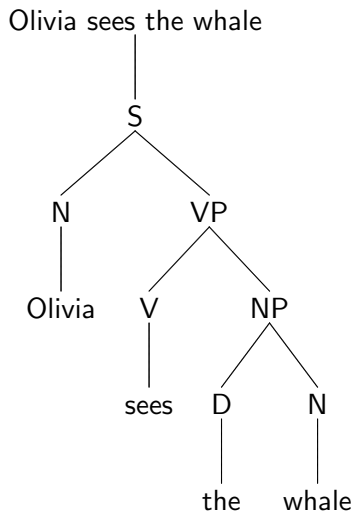
▶ $\sim(\sim P \rightarrow (R \rightarrow Z))$

▶ $\sim(\sim(P)) \rightarrow Q$

▶ $\sim\sim(R \rightarrow (U \rightarrow Q))$

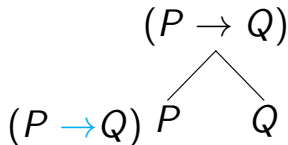
▶ $\sim(\(((S \rightarrow \sim(P \rightarrow \sim Q)) \rightarrow (P \rightarrow \sim Q)) \rightarrow \sim Z)$

Parse trees for natural language



Parsing

- ▶ Any well-formed sentence can be “parsed” into its constituents.
- ▶ Indicate how a sentence is constructed out of its constituents.
- ▶ First locate the **main connective**—this is the last connective introduced when constructing the sentence.



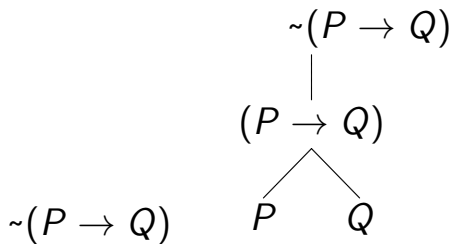
- ▶ Draw branching lines below the main conditional sign and write the antecedent and consequent

Parse trees for \mathcal{L}_1

If the sentence is a negation, the main connective is the negation sign; draw a vertical line under it and write the part of the sentence to which the negation sign is applied.

$$\begin{array}{c} \sim P \\ | \\ \sim P \quad P \end{array}$$

Parse trees for \mathcal{L}_1



Parse trees for \mathcal{L}_1

$(S \rightarrow (P \rightarrow \sim Q))$

