

Subderivations

| University of Edinburgh | PHIL08004 |

$P \rightarrow Q \therefore P \rightarrow ((Q \rightarrow R) \rightarrow R)$

1. Show $P \rightarrow ((Q \rightarrow R) \rightarrow R)$

2.

P

ass cd

3.

Show $(Q \rightarrow R) \rightarrow R$

4.

$Q \rightarrow R$

ass cd

5.

$P \rightarrow Q$

pr1

6.

Q

2,5,mp

7.

R

4,6,mp

8.

7,cd

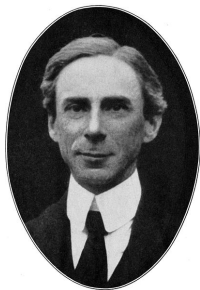
9.

3,cd



A clean-shaven male barber of a certain small town shaves all the inhabitants of the town who do not shave themselves, and never shaves any inhabitant who does shave himself.

Who shaves the barber?



[Bertrand Russell, *The Philosophy of Logical Atomism*]

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$
2. $\neg\neg P$

ass id

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$
2. $\neg\neg P$
3. $\neg T$

ass id

pr1,2,mt

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$
2. $\neg\neg P$ ass id
3. $\neg T$ pr1,2,mt
4. $\neg\neg T$ pr2,2,mt

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$
2. $\neg\neg P$ ass id
3. $\neg T$ pr1,2,mt
4. $\neg\neg T$ pr2,2,mt
5. 3,4,id

$$(T \rightarrow \neg P). (\neg T \rightarrow \neg P). \therefore \neg P$$

1. Show $\neg P$

2. $\neg\neg P$

3. $\neg T$

4. $\neg\neg T$

5.

ass id

pr1,2,mt

pr2,2,mt

3,4,id

$P. \neg(Z \rightarrow P). \therefore \neg Z$

1. Show $\neg Z$

$P. \neg(Z \rightarrow P). \therefore \neg Z$

1. Show $\neg Z$
2. Z

ass id

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z

ass id

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z
3. Show $Z \rightarrow P$

ass id

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4. Z ass cd

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4. Z ass cd
5. P pr1

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4. Z ass cd
5. P pr1
6. 5,cd

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4.

Z

ass cd
5.

P

pr1
6.

--

5,cd

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4.

Z

ass cd
5.

P

pr1
6.

--

5,cd
7. $\neg(Z \rightarrow P)$ pr2

$$P. \neg(Z \rightarrow P). \therefore \neg Z$$

Premise 1 says that P is true, so $Z \rightarrow P$ must also be true. But that contradicts the second premise. And since **ex falso quodlibet**, $\neg Z$.

1. Show $\neg Z$
2. Z ass id
3. Show $Z \rightarrow P$
4.

Z

ass cd
5.

P

pr1
6.

--

5,cd
7. $\neg(Z \rightarrow P)$ pr2

$P. \neg(Z \rightarrow P). \therefore \neg Z$

1. Show $\neg Z$

2. Z ass id

3. Show $Z \rightarrow P$

4. Z ass cd

5. P pr1

6. 5,cd

7. $\neg(Z \rightarrow P)$ pr2

8. 3,7,id

1:62 [T16] $\therefore (\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$

1:91 $\therefore \neg(P \rightarrow \neg P) \rightarrow (\neg P \rightarrow P)$

1:98 pr1. $\neg((U \rightarrow T) \rightarrow \neg Q)$
pr2. $\neg(P \rightarrow (Q \rightarrow R)) \rightarrow (Q \rightarrow S)$
pr3. $(\neg(Q \rightarrow R) \rightarrow \neg P) \rightarrow ((\neg T \rightarrow \neg U) \rightarrow S)$
 $\therefore S$



XO

XO

XO

B

A

-LIFE-

