

Quantifier negation rules

| University of Edinburgh | PHIL08004 |



Puzzle

The Island of Knights and Knaves: *There's an island in which certain inhabitants called "knights" always tell the truth, and others called "knaves" always lie. Every inhabitant of the island is either a knight or a knave.*

Puzzle. Two of the inhabitants—A and B—were standing together in a garden having a beer. A says, "Some inhabitants of this island are not knights". B says "All inhabitants of this island are knights".

The question is, what are A and B?

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The question is, what are A and B?

Some inhabitants
are not knights.

All inhabitants are
knights.

A

B

$$\neg \exists x \neg Hx$$

Apply inference rules to get:

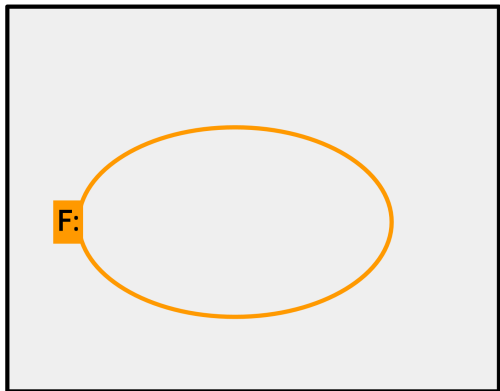
$$Ha$$

$$\neg \exists x \neg Hx$$
$$\forall x Hx$$

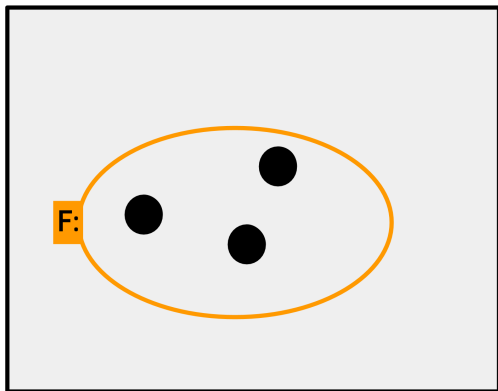
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$$Ha$$

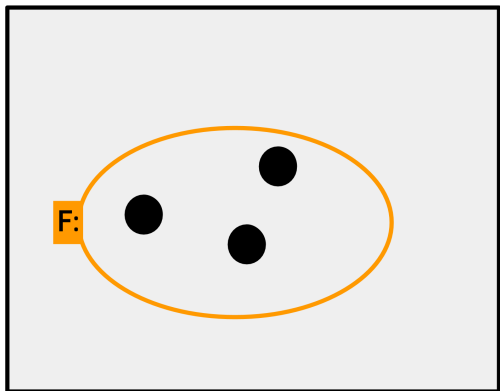
$\neg \exists x \neg Fx$



$$\neg \exists x \neg Fx$$

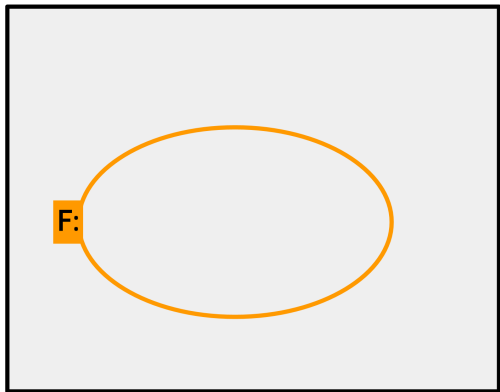


$\neg \exists x \neg Fx$

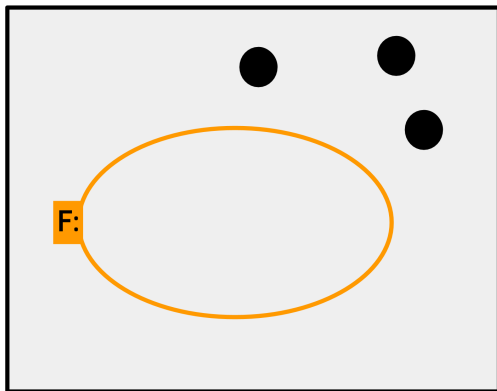


$\forall x Fx$

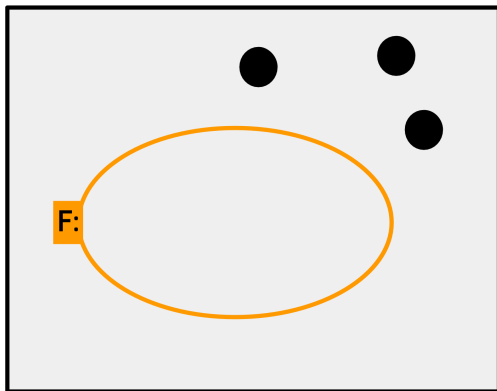
$\neg \exists x Fx$



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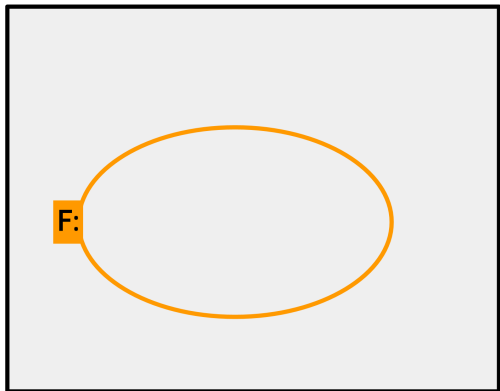


$\neg \exists x Fx$

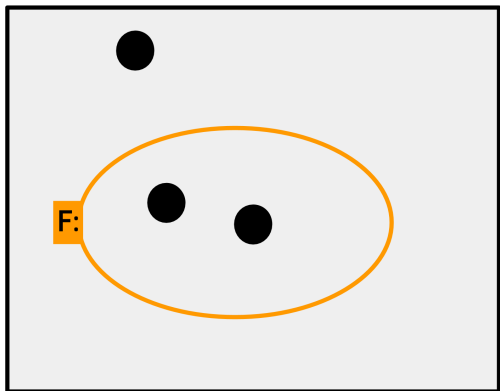


$\forall x \neg Fx$

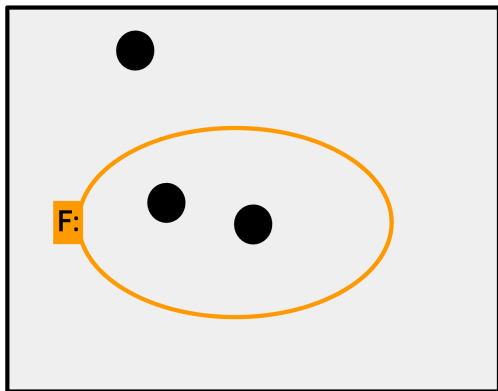
$\neg\forall xFx$



$$\neg \forall x Fx$$

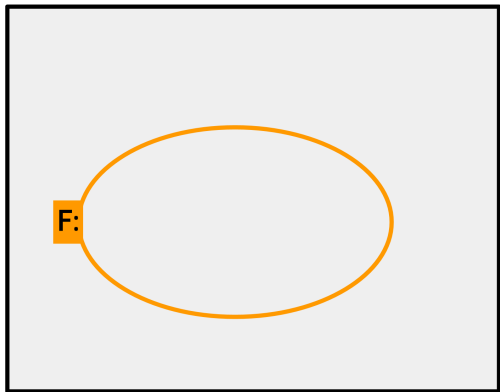


$$\neg \forall x Fx$$

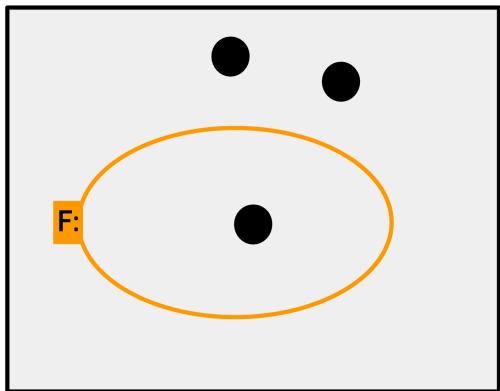


$$\exists x \neg Fx$$

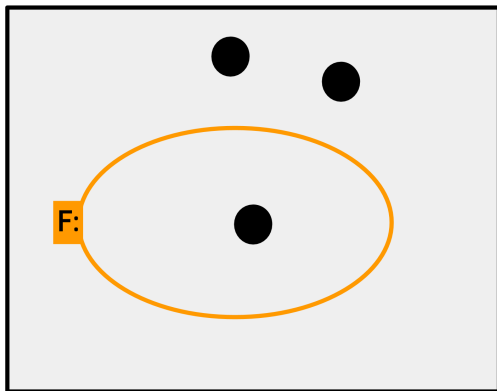
$\neg\forall x\neg Fx$



$$\neg \forall x \neg Fx$$



$$\neg \forall x \neg Fx$$



$$\exists x Fx$$

QUANTIFIER NEGATION RULES (T203-T206)

$$\frac{\neg \forall x \phi}{\exists x \neg \phi} \quad \text{qn}$$

$$\frac{\forall x \phi}{\neg \exists x \neg \phi} \quad \text{qn}$$

$$\frac{\neg \forall x \neg \phi}{\exists x \phi} \quad \text{qn}$$

$$\frac{\forall x \neg \phi}{\neg \exists x \phi} \quad \text{qn}$$

$$\frac{\neg \exists x \phi}{\forall x \neg \phi} \quad \text{qn}$$

$$\frac{\exists x \phi}{\neg \forall x \neg \phi} \quad \text{qn}$$

$$\frac{\neg \exists x \neg \phi}{\forall x \phi} \quad \text{qn}$$

$$\frac{\exists x \neg \phi}{\neg \forall x \phi} \quad \text{qn}$$

$$\neg \forall x Fx \leftrightarrow \exists x \neg Fx$$

1	▼ Show $\neg\forall xFx \leftrightarrow \exists x\neg Fx$	
2	▼ Show $\neg\forall xFx \rightarrow \exists x\neg Fx$	
3	$\neg\forall xFx$	ass, cd
4	▼ Show $\exists x\neg Fx$	
5	$\neg\exists x\neg Fx$	ass, id
6	▼ Show $\forall xFx$	
7	▼ Show Fx	
8	$\neg Fx$	ass, id
9	$\exists x\neg Fx$	8, eg, x, x
10		5, 9, id
11		7, id
12		3, 6, id
13		4, cd
14	▼ Show $\exists x\neg Fx \rightarrow \neg\forall xFx$	
15	$\exists x\neg Fx$	ass, cd
16	▼ Show $\neg\forall xFx$	
17	$\forall xFx$	ass, id
18	$\neg Fz$	15, @, z
19	Fz	17, ui, z
20		18, 19, id
21		16, cd
22	$\neg\forall xFx \leftrightarrow \exists x\neg Fx$	2, 14, cb
23		22, dd