Validity and countermodels

| University of Edinburgh | PHIL08004 |

P	Q	R	$(P \to Q) \land R$	$\neg R \lor P$	Q
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Validity

An argument is tautologically valid if and only if there is no assignment of truth values to its atomic parts which make the premises all true and the conclusion false.

$(P \rightarrow Q) \wedge R. \ \neg R \vee P \therefore Q$

Atomics and rows

- If there is 1 sentence letter, only two rows are required.
- If there are 2 sentence letters, four rows are required.
- If there are 3 sentence letters, eight rows are required.
- ...
- If there are n sentence letters, 2^n rows are required.

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- If there are n sentence letters, 2^n rows are required.

$(P \to Q) \wedge R.\ (\neg R \vee P) \therefore Q$

How many rows?

- n sentence letters = 2^n rows
- 3 sentence letters: P, Q, R
- $2^3 = 2x2x2 = 8$
- 8 rows

$(P \to Q) \land R.\ (\neg R \lor P) \therefore Q$

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$(P \to Q) \wedge R.\ (\neg R \vee P) \therefore Q$ VALID

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$(P \to Q) \land R.\ (\neg R \lor P) \therefore Q$ VALID

P	Q	R	$(P \to Q) \land R$	$\neg R \lor P$	Q
			No row where: premises true + conclusion false		

Checking validity

$$\neg(P \leftrightarrow Q).\ \ R \rightarrow (P \vee Q) \therefore P\ \ \vee \neg R$$

Truth-values analysis

For each of the following either construct a derivation of the conclusion from the premises or show by the method of truth tables that it is invalid.

- $\neg R \rightarrow P.\ \neg S \rightarrow \neg P.\ R \rightarrow S \therefore R$
- $\neg Z.\ (R \rightarrow \neg Z) \rightarrow (Q \wedge P) \therefore (Q \wedge P)$
- $\neg R.\ P \leftrightarrow (R \wedge (P \vee S)) \therefore P \rightarrow \neg S$
- $\neg((P \leftrightarrow Q) \vee \neg(Q \rightarrow P)) \therefore \neg Q \wedge P$

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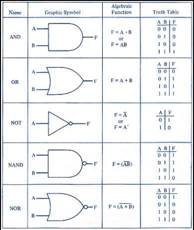

Name	Graphic Symbol	Algebraic Function	Truth Table
AND	A, B → F	F = A · B or F = AB	A B F: 0 0 0; 0 1 0; 1 0 0; 1 1 1
OR	A, B → F	F = A + B	A B F: 0 0 0; 0 1 1; 1 0 1; 1 1 1
NOT	A → F	F = $\bar{A}$ or F = A′	A F: 0 1; 1 0
NAND	A, B → F	F = $(\overline{AB})$	A B F: 0 0 1; 0 1 1; 1 0 1; 1 1 0
NOR	A, B → F	F = $(\overline{A+B})$	A B F: 0 0 1; 0 1 0; 1 0 0; 1 1 0

$$[\![(P \rightarrow Q)]\!]^{v} = \mathrm{T}$$