

\mathcal{L}_2 inference rules

| University of Edinburgh | PHIL08004 |

Conjunction rules:

Rule s (simplification)

$$\begin{array}{l} \Box \wedge \circ \quad \text{or} \quad \Box \wedge \circ \\ \therefore \Box \quad \quad \quad \therefore \circ \end{array}$$

Rule adj (adjunction)

$$\begin{array}{l} \Box \\ \circ \\ \therefore \Box \wedge \circ \end{array}$$

Disjunction rules:

Rule add (addition)

$$\begin{array}{l} \Box \quad \quad \quad \text{or} \quad \quad \quad \Box \\ \therefore \Box \vee \circ \quad \quad \quad \therefore \circ \vee \Box \end{array}$$

Rule mtp (modus tollendo ponens)

$$\begin{array}{l} \Box \vee \circ \quad \quad \text{or} \quad \quad \quad \Box \vee \circ \\ \sim \circ \quad \quad \quad \quad \quad \quad \quad \sim \Box \\ \therefore \Box \quad \quad \quad \quad \quad \quad \quad \therefore \circ \end{array}$$

Biconditional rules:

Rule bc (biconditional-to-conditional)

$$\begin{array}{l} \Box \leftrightarrow \circ \quad \quad \text{or} \quad \quad \quad \Box \leftrightarrow \circ \\ \therefore \Box \rightarrow \circ \quad \quad \quad \therefore \circ \rightarrow \Box \end{array}$$

Rule cb (conditionals-to-biconditional)

$$\begin{array}{l} \Box \rightarrow \circ \\ \circ \rightarrow \Box \\ \therefore \Box \leftrightarrow \circ \end{array}$$

Alfred reads books \therefore Alfred reads books **or** God is an alien.



rule add

Addition: from any sentence you may infer its disjunction with any other sentence.

add:

□

(□ ∨ ○)

(○ ∨ □)

Either Alfred does yoga **or** Elle does yoga. But Alfred doesn't do yoga \therefore Elle does yoga.



rule mtp

Modus tollendo ponens: from a disjunction and the negation of one of its disjuncts you may infer the other disjunct.

mtp:

$$(\square \vee \circ)$$
$$\neg \circ$$

$$\square$$

Alfred reads books and Alfred does yoga. \therefore Alfred does yoga.



rule s

Simplification: if you have a conjunction, you may infer either conjunct.

S:

$(\square \wedge \circ)$

\square

\circ

God is made of spaghetti. God owns a velociraptor.

God is made of spaghetti. God owns a velociraptor.
∴ God is made of spaghetti and owns a velociraptor.



rule adj

Adjunction: if you have any two sentences, you may infer their conjunction, in either order.

adj:

□

○

$(\square \wedge \circ)$

$(\circ \wedge \square)$

Biconditional

- ▶ “James is angry if and only if Ann is happy”
- ▶ ‘if and only if’ is a connective used to combine two propositions
 - ▶ A biconditional says that the truth of either proposition requires the truth of the other, i.e., either both propositions are true, or both are false
- ▶ P **iff** Q
 - ▶ $(P \rightarrow Q)$ and $(Q \rightarrow P)$
- ▶ $(P \leftrightarrow Q)$

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Biconditional

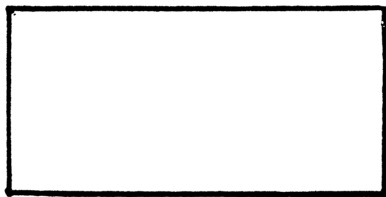
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iff

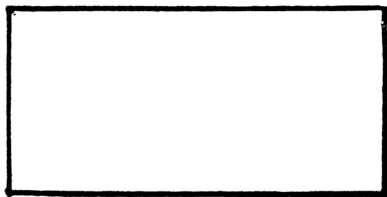
- ▶ x is a rectangle iff:



Rectangle.

iff

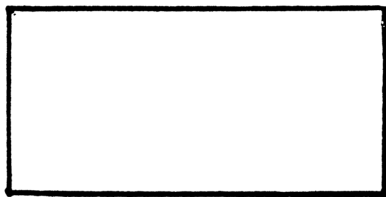
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Rectangle.

iff

- ▶ x is a rectangle iff:
 - ▶ x is a polygon

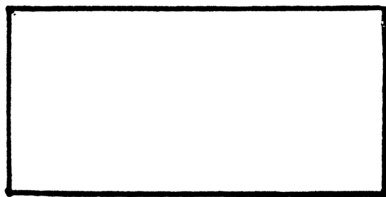


Rectangle.

iff

- ▶ x is a rectangle iff:
 - ▶ x is a polygon

necessary but not sufficient



Rectangle.

iff

- ▶ x is a rectangle iff:
 - ▶ x is a polygon
 - ▶ x has four sides

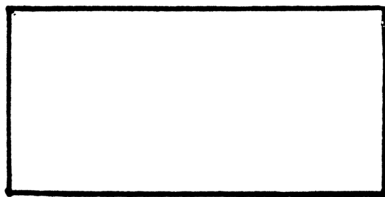
necessary but not sufficient



Rectangle.

iff

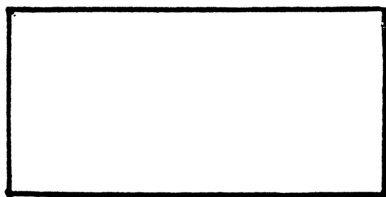
- ▶ x is a rectangle iff:
 - ▶ x is a polygon necessary but not sufficient
 - ▶ x has four sides necessary but not sufficient
 - ▶ x is a quadrilateral with right angles



Rectangle.

iff

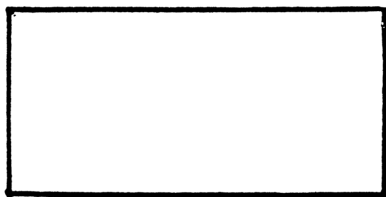
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Rectangle.

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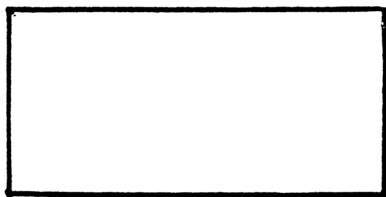
- ▶ x is a rectangle iff:
 - ▶ x is a polygon necessary but not sufficient
 - ▶ x has four sides necessary but not sufficient
 - ▶ x is a quadrilateral with right angles necessary and sufficient
 - ▶ x is a square



Rectangle.

iff

- ▶ x is a rectangle iff:
 - ▶ x is a polygon necessary but not sufficient
 - ▶ x has four sides necessary but not sufficient
 - ▶ x is a quadrilateral with right angles necessary and sufficient
 - ▶ x is a square sufficient but not necessary

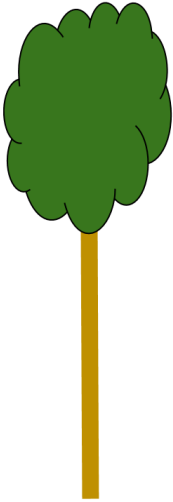
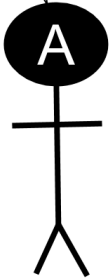


Rectangle.

Puzzle. Two of the inhabitants—A and B—are standing together under a tree. A makes the following statement: “I am a knave iff B is a knave.”

What is B?

"I'm a knave iff B is a knave"



rule bc

Biconditional-to-conditional: from a biconditional you may infer either of the corresponding conditionals.

bc:

$$(\square \leftrightarrow \circ)$$

$$(\square \rightarrow \circ)$$

$$(\circ \rightarrow \square)$$

rule cb

Conditionals-to-biconditional: from two conditionals where the antecedent of one is the consequent of the other, and vice versa, you may infer a biconditional containing the parts of the conditionals.

cb:

$(\square \rightarrow \circ)$

$(\circ \rightarrow \square)$

$(\square \leftrightarrow \circ)$

Practice exercises

2:1 $(P \vee R), (\neg R \wedge T), (P \vee (Q \wedge T)) \rightarrow S \therefore S$

2:3 $(P \leftrightarrow \neg Q) \rightarrow R \therefore (\neg R \wedge P) \rightarrow Q$

2:4 $\neg(\neg R \rightarrow (Q \wedge T)), \neg S \vee R \therefore \neg S$