The language of ‘if’, ‘not’, ‘and’, ‘or’

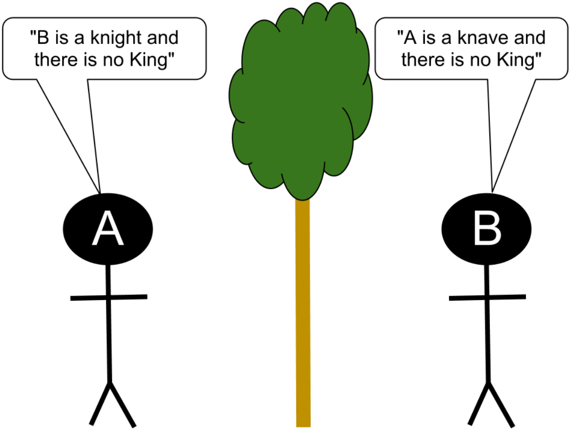
| University of Edinburgh | PHIL08004 |

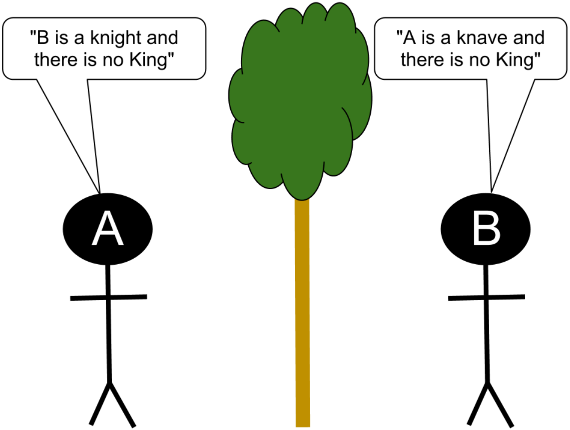
$$(P \wedge Q).\ (R \vee {\sim}Q)\ \therefore R$$

Puzzle. Suppose you are wondering whether or not there is a King of the Island of Knights and Knaves. You come across two inhabitants A and B, who make the following statements:

A: B is a knight and there is no King.
B: A is a knave and there is no King.

Does the island have a King?


"B is a knight and there is no King"
"A is a knave and there is no King"
A
B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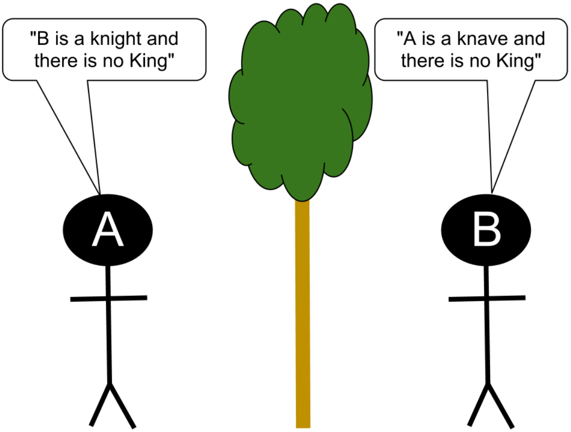
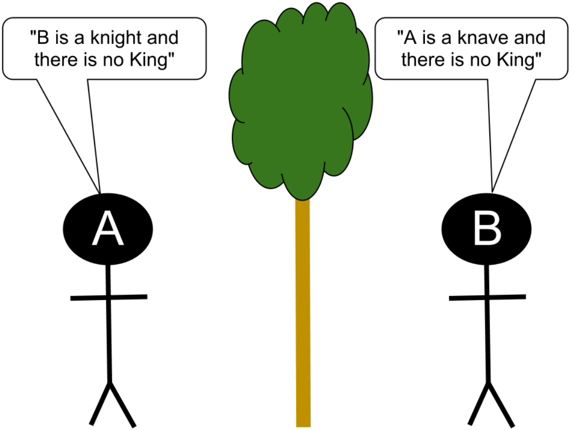
Knight?



Knight?	Knight?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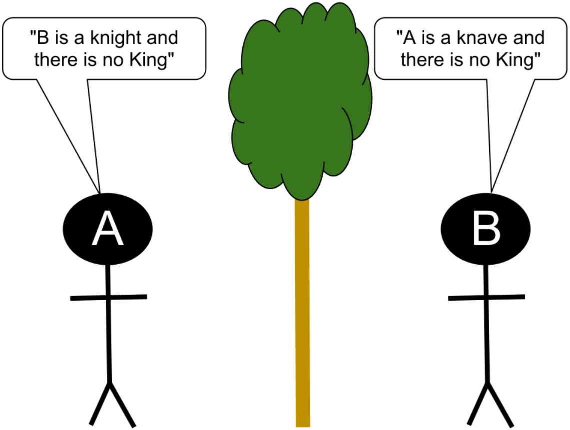
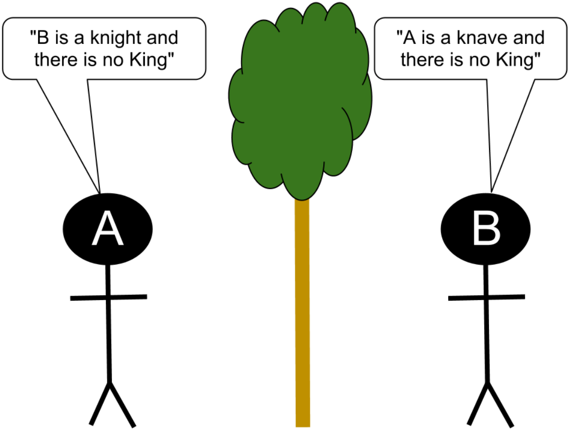


	Knight?
Knave	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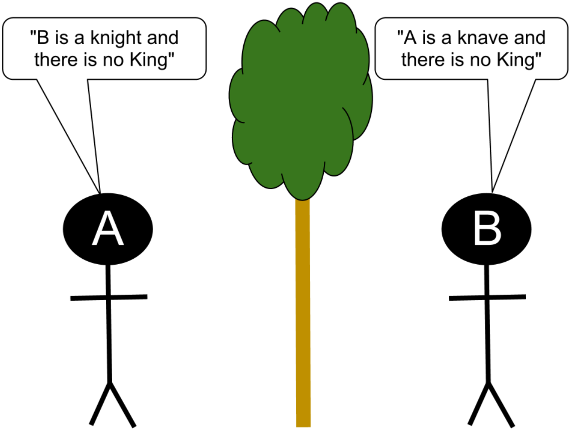
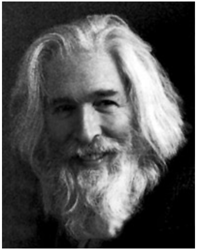


Knave Knave

But is there a king?

YES



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$P \;\therefore Q$

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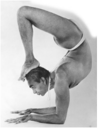

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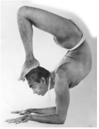

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$\neg(R \rightarrow \neg Y) \therefore Y$

R	Y	$\neg(R \to \neg Y)$	R and Y
T	T		
T	F		
F	T		
F	F		

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T	T	T	
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Functional completeness

- the set of connectives $\{\neg, \rightarrow\}$ is functionally complete
  - other pairs are as well, e.g. $\{\neg, \wedge\}$, $\{\neg, \vee\}$
- for any possible truth-function there is a formula of $\mathscr{L}_1$ that expresses it
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Truth-functional logic

- Sentence letters are assigned **truth-values**
- Truth-values: T, F
- The sentential connectives, $\neg, \rightarrow$, are interpreted as **functions**
  - they map truth-values to truth-values

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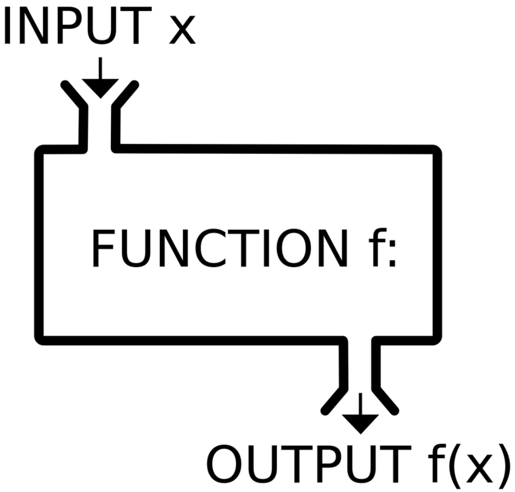

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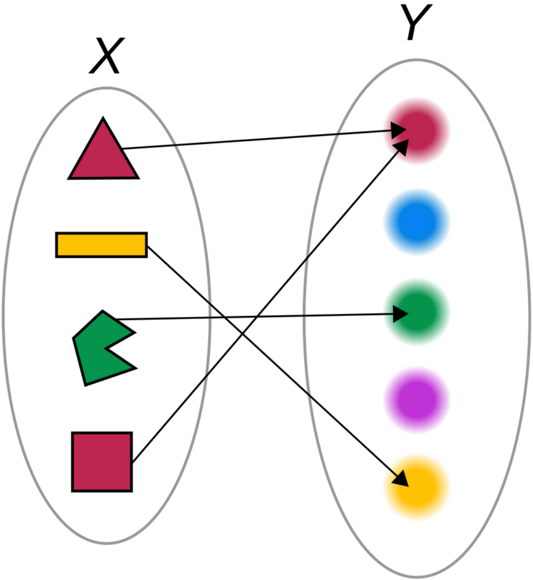
- Function: an input type, an output type, and a special mapping.


X
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- Outputs: $\{T, F\}$

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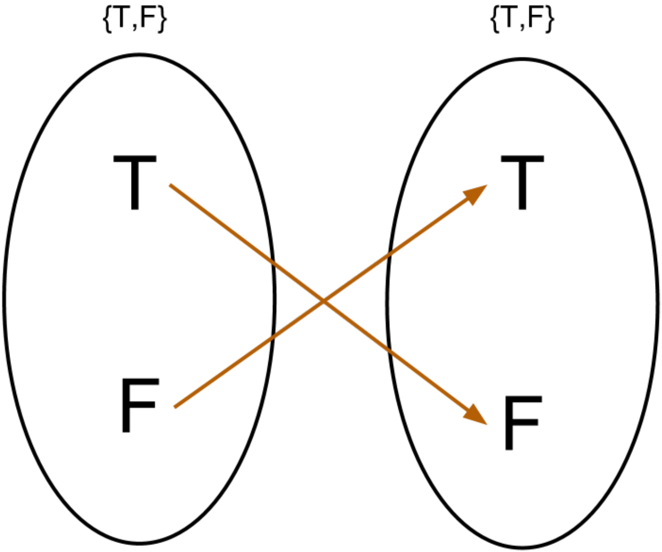
{T,F}
{T,F}
T
F
T
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Truth functions

- Inputs: $\{(T,T),(T,F),(F,T),(F,F)\}$
- Outputs: $\{T,F\}$

- $h(T,T) = T$
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‘and’, ‘or’

- Alfred either read a book or Alfred did yoga. Alfred didn’t do yoga. So Alfred read a book.
- Alfred is smart. Alfred is overconfident. So, Alfred is smart but overconfident.

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- Basic symbols:
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- "James is angry and Ann is happy"
- 'and' is a connective used to combine two propositions
    - A conjunction says that both of the propositions are true
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    - P, Q are **conjuncts**
- $(P \wedge Q)$

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- 'and' is a connective used to combine two propositions
  - A conjunction says that both of the propositions are true
- P and Q
  - P, Q are **conjuncts**
- $(P \wedge Q)$

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∧

- Truth table for conjunction

$\square$	$\bigcirc$	$(\square \wedge \bigcirc)$
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T	F	F
F	T	F
F	F	F

$\wedge$

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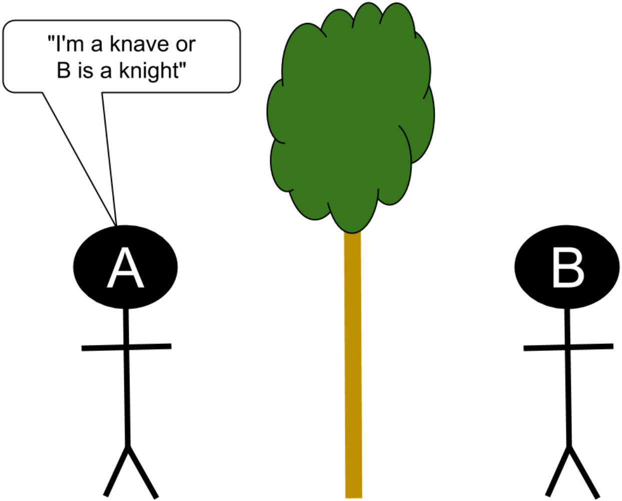
Puzzle. Two of the inhabitants—A and B—are standing together under a tree. A makes the following statement: "Either I am a knave or B is a knight."

What are A and B?


"I'm a knave or
B is a knight"
A
B

OR

- **Exclusive OR**: "I'll either marry Elle or Ruby (but not both)"
- **Inclusive OR**: "Must either pass a logic course or a foreign language course (or both)"

P	Q	P XOR Q
true	*true*	*false*
true	*false*	*true*
false	*true*	*true*
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P	Q	$P \vee Q$
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Disjunction

- "James is angry or Ann is happy"
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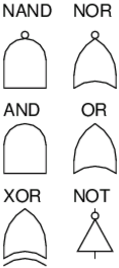
NAND
NOR
AND
OR
XOR
NOT