Answer the following questions on separate sheets of paper. Your answers should be precise and your handwriting should be clear.

Symbolisations. *Construct symbolisations—logically perspicuous symbolisations—for each of the following sentences or arguments in the language of predicate logic $\mathcal{L}_3$. Indicate in each case the scheme of abbreviation used.*

1. If Alfred loves contradictions, then he both loves and doesn't love contradictions.

2. If all monkeys have a banana and George is a monkey, then George has a banana.

3. Either all students pass or some students are sad.

4. Some mushrooms are poisonous but all of Timothy's mushrooms are magical.

5. Some aliens are people. Therefore, it is not the case that all people are human.

6. All aliens who are green are mean but some mean aliens are not green.

7. If everything is mental, then nothing is physical unless something is both mental and physical.

Derivations. *Show by constructing annotated derivations that the following arguments are valid. In addition to the basic rules you may also use any of the derived rules.*

8. $(P \vee Q)$. $\neg P$. $(S \to P)$ $\therefore$ $(\neg S \wedge Q)$

9. $\therefore (P \leftrightarrow Q) \to \neg(P \wedge \neg Q)$

10. $((\neg Q \wedge T) \to \neg S)$. T $\therefore$ $(\neg S \vee Q)$

11. $(T \vee (S \to R)) \vee (\neg S \vee Q)$ $\therefore$ $(P \to Q) \vee ((R \vee \neg S) \vee T)$

12. $\exists x Fx$ $\therefore \exists x(Hx \to Fx)$

13. $\forall x(Gx \leftrightarrow \forall z Hz)$ $\exists y Gy \therefore \forall x Gx$

14. $\exists y \forall x(Fx \leftrightarrow Fy)$. $\exists x Fx$ $\therefore \forall x Fx$

15. $\exists x(Fx \to Q)$. $\exists x(Q \to Fx)$ $\therefore \exists x(Fx \leftrightarrow Q)$

16. $\forall y(Dy \to My)$. $\neg \exists x(Hx \wedge \neg Dx)$. $\therefore Hc \to \exists x(Mx \wedge Dx)$

17. $\therefore$ $\forall x((Fx \wedge (\neg \exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$

Countermodels and validity. *For each of the following arguments either construct a derivation of the conclusion from the premises or show that it is invalid by constructing a relevant countermodel.*

18. $(\neg R \to P)$. $(\neg S \to \neg P)$. $(R \to S) \therefore R$

19. $(\neg P \to Q)$. $(P \to Q) \therefore Q$

20. $\exists x(Fx \to P) \quad \therefore (\exists x Fx \to P)$

21. $(\exists x Fx \to P) \quad \therefore \exists x(Fx \to P)$

22. $\exists x(Fx \wedge Gx)$. $\exists x(Fx \wedge Hx)$. $\neg \exists x(Gx \wedge Hx) \ \therefore \forall x Fx$

23. $(\exists y Ly \to \forall x Fx)$. $(\forall z(Fz \vee Dz) \to \forall x Mx) \quad \therefore \ \forall x(\neg Lx \vee Mx)$

24. $\exists y Hy$. $(Ha \wedge Hb) \ \therefore \ \forall y Hy$

25. $\therefore \neg(\exists x \neg Fx \wedge (Fa \wedge Fb))$

26. $\forall x \exists y(Fx \leftrightarrow (Gy \vee Fx)) \therefore (\neg \exists x Fx \to \neg \exists x Gx)$

27. Descartes is conscious. If something is not physical, then something is not explained by the natural sciences. If something is conscious, then something is not explained by the natural sciences. $\therefore$ Everything is physical.

28. A Puzzle in the Land of Inquiry

Somewhere in the vast reaches of space, there is strange land known as *the Land of Inquiry*. The name derives from that fact that its inhabitants only ask questions—they never make assertions. The inhabitants only ask yes-no questions. Each inhabitant is one of two types *Yesies* and *Noies*. Yesies ask only questions whose correct answer is 'yes'; Noies ask only questions whose correct answer is 'no'.

For example, a Yesie could ask, "Does two plus two equal four?" But she could not ask whether two plus two equals five. And a Noie could not ask whether two plus two equals four, but she could ask "Does two plus two equal five?"

Question: If in the Land of Inquiry you meet Mr and Mrs Tarski, and Mr Tarski asks you "Are Mrs Tarski and I both Noies?", what should you say?

29. Is your answer to question (29) 'no'?

(a) no

(b) yes

Advanced problems. (NOTE: These are for only for students trying to get close to 100 on the exam. In addition, one must read chapter 4 and do all the exercises therein.) *For each of the following arguments either construct a derivation of the conclusion from the premises or show that it is invalid by constructing a relevant countermodel.*

30. $\forall x \forall y(Gxy \to Gyx)$. $\exists x Gax$ $\therefore$ $\exists x(Gax \land \exists y Gxy)$

31. $\therefore$ $\neg \exists x(Fx \land Rxa) \leftrightarrow \forall x(Fx \to \neg Rxa)$

32. $\forall x(Fx \to \exists y(Gy \land Rxy))$ $\therefore$ $\forall x(Gx \to \exists y(Fy \land Ryx))$

33. $\forall x \forall y \exists z((Fxy \land Fyz) \to Fxz)$. $\forall x Fxx$ $\therefore$ $\exists x \forall y(Fxy \to Fyx)$