Practice Exam Answers

Logic 1: Rabern

symbolisations

1. F: loves contradictions
   a: Alfred

   $(Fa \rightarrow (Fa \wedge \sim Fa))$

2. F: is a monkey
   G: has a banana
   a: George

   $((\forall x(Fx \rightarrow Gx) \wedge Fa) \rightarrow Ga)$

3. F: is a student
   G: passes
   H: is sad

   $(\forall x(Fx \rightarrow Gx) \vee \exists x(Fx \wedge Hx))$

4. F: is a mushroom
   G: is poisonous
   H: is owned by Timothy
   M: is magical

   $(\exists x(Fx \wedge Gx) \wedge \forall x((Hx \wedge Fx) \rightarrow Mx))$

5. F: is an alien
   G: is a person
   H: is human

   $\exists x(Fx \wedge Gx) \quad \therefore \sim \forall x(Gx \rightarrow Hx)$

6. F: is an alien
G: is green
H: is mean

$$\left(\forall x\left((Fx \wedge Gx) \rightarrow Hx\right) \wedge \exists x\left((Hx \wedge Fx) \wedge \sim Gx\right)\right)$$

7. F: is mental
G: is physical

$$\left(\forall x Fx \rightarrow \left(\sim \exists x Gx \vee \exists x(Fx \wedge Gx)\right)\right)$$

Derivations

8. $P \vee Q.\ \sim P.\ (S \rightarrow P)\ \therefore\ (\sim S \wedge Q)$

1.	~~Show~~ $(\sim S \wedge Q)$	
2.	Q	pr1, pr2, mtp
3.	$\sim S$	pr2, pr3, mt
4.	$\sim S \wedge Q$	2, 3, adj
5.		4, dd

9. $\therefore\ (P \leftrightarrow Q) \rightarrow \sim(P \wedge \sim Q)$

1.	~~Show~~ $(P \leftrightarrow Q) \rightarrow \sim(P \wedge \sim Q)$	
2.	$P \leftrightarrow Q$	ass cd
3.	~~Show~~ $\sim(P \wedge \sim Q)$	
4.	$P \wedge \sim Q$	ass id
5.	$P \rightarrow Q$	2, bc
6.	P	4, s
7.	Q	5, 6, mp
8.	$\sim Q$	4, s
9.		7, 8, id
10.		3, cd

10. $(\sim Q \wedge T) \rightarrow \sim S.\quad T \quad \therefore\ \sim S \vee Q$

1.	~~Show~~ $\sim S \vee Q$	
2.	$\sim(\sim S \vee Q)$	ass id
3.	$\sim\sim S \wedge \sim Q$	2, dm
4.	$\sim\sim S$	3, s
5.	$\sim(\sim Q \wedge T)$	4, pr1, mt
6.	$\sim\sim Q \vee \sim T$	5, dm
7.	$\sim Q$	3, S
8.	$\sim\sim\sim Q$	7, dn
9.	$\sim T$	6, 8, mtp
10.	T	pr2
11.		9, 10, id

11. $(T \vee (S \rightarrow R)) \vee (\sim S \vee Q) \quad \therefore\ (P \rightarrow Q) \vee ((R \vee \sim S) \vee T)$

1.	~~Show~~ $(P \rightarrow Q) \vee ((R \vee \sim S) \vee T)$	
2.	$\sim((P \rightarrow Q) \vee ((R \vee \sim S) \vee T))$	ass id
3.	$\sim(P \rightarrow Q) \wedge \sim((R \vee \sim S) \vee T)$	2, dm
4.	$\sim((R \vee \sim S) \vee T)$	3, s
5.	$\sim(R \vee \sim S) \wedge \sim T$	4, dm
6.	$\sim R \wedge \sim\sim S$	5, s, dm
7.	$\sim\sim S$	6, s
8.	$\sim R$	6, s
9.	$S \wedge \sim R$	7, dn, 8, adj
10.	$\sim(S \rightarrow R)$	9, nc
11.	$\sim T \wedge \sim(S \rightarrow R)$	5, S, 10, adj
12.	$\sim(T \vee (S \rightarrow R))$	11, dm
13.	$\sim S \vee Q$	12, pr, mtp
14.	Q	7, 13, mtp
15.	$P \wedge \sim Q$	3, s, nc
16.	$\sim Q$	15, s
17.		16, 14, id

12. $\exists x Fx \quad \therefore \quad \exists x(Hx \to Fx)$

1.	~~Show~~ $\exists x(Hx \to Fx)$	
2.	Fz	
3.	~~Show~~ $Hz \to Fz$	
4.	Fz	2, r
5.		4, cd
6.	$\exists x(Hx \to Fx)$	3, eg
7.		6, dd

13. $\forall x(Gx \leftrightarrow \forall z Hz), \; \exists y Gy \quad \therefore \quad \forall x Gx$

1.	~~Show~~ $\forall x Gx$	
2.	$\sim \forall x Gx$	ass id
3.	$\exists x \sim Gx$	2, qn
4.	$\sim Gu$	3, ei
5.	Gw	pr2, ei
6.	$Gu \leftrightarrow \forall z Hz$	pr1, ui
7.	$Gw \leftrightarrow \forall z Hz$	pr1, ui
8.	$\forall z Hz \to Gu$	6, bc
9.	$Gw \to \forall z Hz$	7, bc
10.	$\sim \forall z Hz$	4, 8, mt
11.	$\forall z Hz$	5, 9, mp
12.		10, 11, id

14. $\exists y \forall x (Fx \leftrightarrow Fy)$. $\exists x Fx$ $\therefore \forall x Fx$

1.	~~Show~~ $\forall x Fx$	
2.	$\sim \forall x Fx$	ass id
3.	$\exists x \sim Fx$	2, qn
4.	$\sim Fu$	3, ei
5.	Fz	pr2, ei
6.	$\forall x (Fx \leftrightarrow Fw)$	pr1, ei
7.	$Fu \leftrightarrow Fw$	6, ui
8.	$Fz \leftrightarrow Fw$	6, ui
9.	$Fw \rightarrow Fu$	7, bc
10.	$Fz \rightarrow Fw$	8, bc
11.	$\sim Fw$	4, 9, mt
12.	Fw	5, 10, mp
13.		11, 12, id

15. $\exists x (Fx \rightarrow Q)$. $\exists x (Q \rightarrow Fx)$ $\therefore \exists x (Fx \leftrightarrow Q)$

1.	~~Show~~ $\exists x (Fx \leftrightarrow Q)$	
2.	$\sim \exists x (Fx \leftrightarrow Q)$	ass id
3.	$\forall x \sim (Fx \leftrightarrow Q)$	2, qn
4.	$Fz \rightarrow Q$	pr1, ei
5.	$Q \rightarrow Fu$	pr2, ei
6.	$\sim (Fz \leftrightarrow Q)$	3, ui
7.	$Fz \leftrightarrow \sim Q$	6, nb
8.	$Fu \leftrightarrow \sim Q$	3, ui, nb
9.	~~Show~~ $Q \rightarrow Fz$	
10.	Q	ass cd
11.	Fu	10, 5, mp
12.	$\sim Q$	8, bc, 11, mp
13.		10, 12, id
14.	$Fz \leftrightarrow Q$	4, 9, cb
		6, 14, id

$\exists x(Fx \to Q).\ \exists x(Q \to Fx)\ \therefore\ \exists x(Fx \leftrightarrow Q)$

15\. Alternative derivation using sc ("seperation of cases")

$$\text{sc:}\quad \frac{\begin{array}{c}\Box \to \Delta \\ \sim\Box \to \Delta\end{array}}{\Delta}$$

1.	~~Show~~ $\exists x(Fx \leftrightarrow Q)$	
2.	~~Show~~ $Q \to \exists x(Fx \leftrightarrow Q)$	
3.	Q	ass cd
4.	~~Show~~ $\exists x(Fx \leftrightarrow Q)$	
5.	$\sim\exists x(Fx \leftrightarrow Q)$	ass id
6.	$\forall x \sim(Fx \leftrightarrow Q)$	5, qn
7.	$Q \to Fz$	pr2, ei
8.	Fz	3, 7, mp
9.	$\sim(Fz \leftrightarrow Q)$	6, ui
10.	$Fz \to \sim Q$	9, nb, bc
11.	$\sim Q$	8, 10, mp
12.	Q	3, r
13.		11, 12, id
14.		4, cd
15.	~~Show~~ $\sim Q \to \exists x(Fx \leftrightarrow Q)$	
16.	$\sim Q$	ass. cd
17.	~~Show~~ $\exists x(Fx \leftrightarrow Q)$	
18.	$\sim\exists x(Fx \leftrightarrow Q)$	ass id
19.	$\forall x \sim(Fx \leftrightarrow Q)$	18, qn
20.	$Fu \to Q$	pr1, ei
21.	$\sim Fu$	16, 20, mt
22.	$\sim(Fu \leftrightarrow Q)$	19, ui
23.	$\sim Q \to Fu$	22, nb, bc
24.	Fu	16, 23, mp
25.		21, 24, id
26.		17, cd
27.	$\exists x(Fx \leftrightarrow Q)$	2, 15, sc, dd

16. $\forall y(Dy \to My).\ \sim\exists x(Hx \wedge \sim Dx).\ \therefore Hz \to \exists x(Mx \wedge Dx)$

1.	~~show~~ $Hz \to \exists x(Mx \wedge Dx)$	
2.	Hz	ass cd
3.	~~show~~ $\exists x(Mx \wedge Dx)$	
4.	$\sim\exists x(Mx \wedge Dx)$	ass id
5.	$\forall x \sim(Mx \wedge Dx)$	4, qn
6.	$\sim(Mz \wedge Dz)$	5, ui
7.	$\sim(Hz \wedge \sim Dz)$	pr2, qn, ui
8.	$Dz \to Mz$	pr1, ui
9.	$\sim Hz \vee \sim\sim Dz$	7, dm
10.	$\sim\sim Hz$	2, dn
11.	$\sim\sim Dz$	10, 9, mtp
12.	Dz	11, dn
13.	Mz	12, 8, mp
14.	$\sim Mz \vee \sim Dz$	6, dm
15.	$\sim\sim Mz$	13, dn
16.	$\sim Dz$	14, 15, mtp
17.		16, 12, id
18.		3, cd

17. $\therefore \forall x((Fx \wedge (\sim\exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$

1.	~~show~~ $\forall x((Fx \wedge (\sim\exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$	
2.	$\sim\forall x((Fx \wedge (\sim\exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$	ass id
3.	$\exists x \sim((Fx \wedge (\sim\exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$	2, qn
4.	$\sim((Fz \wedge (\sim\exists x Fx \vee \forall x Gx)) \to \forall x(Fx \vee Gx))$	3, ei
5.	$(Fz \wedge (\sim\exists x Fx \vee \forall x Gx)) \wedge \sim\forall x(Fx \vee Gx)$	4, nc
6.	$\exists x \sim(Fx \vee Gx)$	5, s, qn
7.	$\sim(Fw \vee Gw)$	6, ei
8.	$\sim Fw \wedge \sim Gw$	7, dm
9.	$\sim Gw$	8, s
10.	$\exists x \sim Gx$	9, eg
11.	$\sim\forall x Gx$	10, qn
12.	$\sim\exists x Fx \vee \forall x Gx$	5, s, s
13.	$\sim\exists x Fx$	11, 12, mtp
14.	$\forall x \sim Fx$	13, qn
15.	$\sim Fz$	14, ui
16.	Fz	5, s, s
17.		15, 16, id

18. INVALID

R: F
P: T
S: T

19. VALID

$(\sim P \to Q).\ (P \to Q)\ \therefore\ Q$

1.	~~Show~~ Q	
2.	~ Q	ass id
3.	~ P	2, pr2, mt
4.	Q	3, pr1, mp
5.		2, 4, id

20. INVALID

U: {0, 1}
F: {0}
P: F

21. Valid [Derivation omitted]

22. Invalid

U: {0,1,2}
F: {1,2}
G: {1}
H: {2}

23. Valid [Derivation omitted]

24. Invalid

U: {0,1}
H: {1}
a: 1
b: 1

25. Invalid

U: {0,1}
F: {1}
a: 1
b: 1

26. Invalid [Countermodel omitted]

27. Invalid [Countermodel omitted]

28. [Answer omitted]

29. **head explosion** (See the “Liar Paradox”)