

**Symbolisations.** *Construct symbolisations—logically perspicuous symbolisations—for each of the following sentences or arguments in the language  $L_2$ . Indicate in each case the scheme of abbreviation used.*

0. If Alfred loves tautologies, then he loves tautologies.
1. Alfred slept through lecture, but he passed.
2. If Jane doesn't go to the party, then Alfred and David will both go to the party.
3. Either Jane buys beer and Tom doesn't buy beer or Tom buys beer and Aidan doesn't.
4. If Jane likes the wine just in case it is red, then she likes Pinot noir but she likes neither Chardonnay nor Riesling.
5. If Alfred sleeps in he misses class. If he doesn't sleep in he is tired. If he misses class, he is not tired. So, Alfred sleeps in.
6. If Descartes can doubt that he is thinking, then he thinks. If Descartes cannot doubt that he is thinking, then he thinks. If Descartes does not exist, then he does not think. Thus, Descartes exists.

**Derivations.** *Show by constructing annotated derivations that the following arguments are valid. In addition to the basic rules you may only use the following derived rules:  $nc$ ,  $cdj$ ,  $sc$ ,  $dm$ ,  $nb$ .*

7.  $\neg Q \therefore ((P \rightarrow Q) \rightarrow \neg P)$
8.  $(P \rightarrow (Q \rightarrow R)). (P \rightarrow (R \rightarrow S)) \therefore (P \rightarrow (Q \rightarrow S))$
9.  $(\neg P \rightarrow Q). (P \rightarrow Q) \therefore Q$
10.  $\neg(R \rightarrow Q). Q \therefore (P \rightarrow S)$
11.  $((R \rightarrow \neg S) \rightarrow (P \wedge Q)). \neg S \therefore (P \wedge Q)$
12.  $(R \vee P). \neg P. (S \rightarrow P) \therefore (R \wedge \neg S)$
13.  $((P \vee R) \rightarrow Q). ((Q \wedge R) \rightarrow P). R \therefore (P \leftrightarrow Q)$
14.  $(W \vee R). (W \rightarrow \neg(Q \rightarrow \neg P)) \therefore (P \vee R)$
15.  $\neg((P \wedge R) \rightarrow Q) \therefore ((P \wedge R) \wedge \neg Q)$
16.  $\neg(P \vee (Q \rightarrow \neg Z)). (Z \vee Q) \therefore \neg(Q \rightarrow \neg Z)$

17.  $\therefore (P \leftrightarrow Q) \rightarrow \neg(P \wedge \neg Q)$
18.  $((\neg Q \wedge T) \rightarrow \neg S). T \therefore (\neg S \vee Q)$
19.  $((T \vee (S \rightarrow R)) \vee (\neg S \vee Q)) \therefore ((P \rightarrow Q) \vee ((R \vee \neg S) \vee T))$
20.  $\neg(P \rightarrow Q). Q \therefore (R \rightarrow Z)$
21.  $(\neg R \vee (P \leftrightarrow \neg S)). (\neg(\neg R \rightarrow P) \rightarrow S) \therefore (P \vee S)$
22.  $((Z \wedge R) \rightarrow P). ((P \vee R) \rightarrow Z). R \therefore (P \leftrightarrow Z)$
23.  $((Q \wedge S) \rightarrow R). (Q \vee P). (\neg P \vee Q). S \therefore R$

*Prove the following theorems without using either nc or dm:*

24.  $\therefore (Q \wedge \neg R) \rightarrow \neg(Q \rightarrow R)$
25.  $\therefore \neg(Z \wedge R) \rightarrow (\neg Z \vee \neg R)$
26.  $\therefore \neg(\neg P \vee \neg Q) \rightarrow P$
27.  $\therefore \neg(S \rightarrow X) \rightarrow \neg X$
28.  $\therefore (P \vee \neg P)$

29. Assume the sentential connective ‘ $\downarrow$ ’ has the semantics provided in the truth-table below—what formula of our language L2 would be logically equivalent to ‘ $(P \downarrow Q)$ ’?

$\square$	$\circ$	$(\square \downarrow \circ)$
$T$	$T$	$F$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$T$

30. You visit the island of knights and knaves—where “knights” always tell the truth, and “knaves” always lie, and every inhabitant of the island is either a knight or a knave. Let’s say that two inhabitants are the same *type* of inhabitant, if they are both knights or both knaves. You come across three inhabitants: A, B, and C. A says “B and C are the same type of inhabitant”. Someone then asks C, “Are A and B of the same type of inhabitant?” What does C answer? Why?