MIDTERM EXAM [Logic 1: Rabern]
University of Edinburgh Name:

Answer the following questions on separate sheets of paper. Your answers should be precise and your handwriting should be clear.

Symbolisations. *Construct symbolisations—logically perspicuous symbolisations—for each of the following sentences or arguments in the language L2. Indicate in each case the scheme of abbreviation used.*

0. If Alfred loves contradictions, then he both loves and doesn't love contradictions.

1. If Jane attends the party, then Frank won't attend if Alfred does.

2. Either Frank buys wine and Adi buys cheese or Frank doesn't buy wine.

3. If Descartes can doubt that he is thinking, then he thinks. If Descartes cannot doubt that he is thinking, then he thinks. If Descartes does not exist, then he does not think. Thus, Descartes exists.

Derivations. *Show by constructing annotated derivations that the following arguments are valid. In addition to the basic rules you may only use the following derived rules:* `nc`, `cdj`, `sc`, `dm`, `nb`.

4. $(\neg R \to \neg W).\ (R \to \neg W) \therefore \neg W$

5. $(Z \to (Q \to R)).\ (Z \to (R \to P)) \therefore (Z \to (Q \to P))$

6. $\neg(Q \to R).\ \ (R \wedge (\neg W \vee S))\ \ \therefore (P \to S)$

7. $((Z \wedge R) \to P).\ ((P \vee R) \to Z).\ R\ \therefore (P \leftrightarrow Z)$

Prove the following theorem using neither `nc` *nor* `dm`:

8. $\therefore \neg(Q \to R) \to (Q \wedge \neg R)$

Semantics.

9. Assume the sentential connective '+' has the semantics provided in the truth-table below—what formula of our language L2 would be logically equivalent to '$(P + Q)$'?

$\square$	$\bigcirc$	$(\square + \bigcirc)$
T	T	F
T	F	T
F	T	T
F	F	F

10. Three gods A, B, and C are called, in no particular order, 'True', 'False', and 'Random'. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for 'yes' and 'no' are 'da' and 'ja', in some order. You do not know which word means which.

Clarifications:

- It could be that some god gets asked more than one question (and hence that some god is not asked any question at all).
- What the second question is, and to which god it is put, may depend on the answer to the first question. (And of course similarly for the third question.)
- Whether Random speaks truly or not should be thought of as depending on the flip of a coin hidden in his brain: if the coin comes down heads, he speaks truly; if tails, falsely.
- Random will answer 'da' or 'ja' when asked any yes-no question.

$$\begin{array}{l}(\phi \to \psi)\\ \phi\\ \hline \psi\end{array}\ \textbf{mp}$$

$$\begin{array}{l}\phi\\ \hline \neg\neg\phi\end{array}\ \textbf{dn}$$

$$\begin{array}{l}(\phi \to \psi)\\ \neg\psi\\ \hline \neg\phi\end{array}\ \textbf{mt}$$

$$\begin{array}{l}\phi\\ \hline \phi\end{array}\ \textbf{r}$$

$$\begin{array}{l}(\phi \wedge \psi)\\ \hline \phi\end{array}\ \textbf{s}$$

$$\begin{array}{l}\phi\\ \psi\\ \hline (\phi \wedge \psi)\end{array}\ \textbf{adj}$$

$$\begin{array}{l}\phi\\ \hline (\phi \vee \psi)\end{array}\ \textbf{add}$$

$$\begin{array}{l}(\phi \vee \psi)\\ \neg\phi\\ \hline \psi\end{array}\ \textbf{mtp}$$

$$\begin{array}{l}(\phi \to \psi)\\ (\psi \to \phi)\\ \hline (\phi \leftrightarrow \psi)\end{array}\ \textbf{cb}$$

$$\begin{array}{l}(\phi \leftrightarrow \psi)\\ \hline (\phi \to \psi)\\ (\psi \to \phi)\end{array}\ \textbf{bc}$$