$$\frac{(\phi \rightarrow \psi) \quad \phi}{\psi}\ \textbf{mp}$$

$$\frac{\phi}{\neg\neg\phi}\ \textbf{dn}$$

$$\frac{(\phi \rightarrow \psi) \quad \neg\psi}{\neg\phi}\ \textbf{mt}$$

$$\frac{\phi}{\phi}\ \textbf{r}$$

$$\frac{(\phi \wedge \psi)}{\phi}\ \textbf{s}$$

$$\frac{\phi \quad \psi}{(\phi \wedge \psi)}\ \textbf{adj}$$

$$\frac{\phi}{(\phi \vee \psi)}\ \textbf{add}$$

$$\frac{(\phi \vee \psi) \quad \neg\phi}{\psi}\ \textbf{mtp}$$

$$\frac{(\phi \rightarrow \psi) \quad (\psi \rightarrow \phi)}{(\phi \leftrightarrow \psi)}\ \textbf{cb}$$

$$\frac{(\phi \leftrightarrow \psi)}{(\phi \rightarrow \psi) \quad (\psi \rightarrow \phi)}\ \textbf{bc}$$

UNIVERSAL INSTANTIATION

$$\frac{\forall\alpha\phi_\alpha}{\phi_\beta}\ \textbf{ui}$$

Provided that α is a variable, β is a name or variable, and ϕ_β comes from ϕ_α by proper substitution of β for α.

EXISTENTIAL GENERALIZATION

$$\frac{\phi_\beta}{\exists\alpha\phi_\alpha}\ \textbf{eg}$$

Provided that α is a variable, β is a name or variable, and ϕ_β comes from ϕ_α by proper substitution of β for α.

EXISTENTIAL INSTANTIATION

$$\frac{\exists\alpha\phi_\alpha}{\phi_\beta}\ \textbf{ei}$$

Provided that α is a variable, β is a *new* variable, and ϕ_β comes from ϕ_α by proper substitution of β for α.

Negation of conditional (T40)

$$\frac{\neg(\phi \to \psi)}{(\phi \wedge \neg\psi)}\ \textbf{nc}$$

$$\frac{(\phi \wedge \neg\psi)}{\neg(\phi \to \psi)}\ \textbf{nc}$$

Conditional as disjunction (T45, T46)

$$\frac{(\phi \to \psi)}{(\neg\phi \vee \psi)}\ \textbf{cdj}$$

$$\frac{(\neg\phi \vee \psi)}{(\phi \to \psi)}\ \textbf{cdj}$$

Separation of cases (T33, T49)

$$\frac{\begin{array}{l}(\phi \vee \psi)\\ (\phi \to \chi)\\ (\psi \to \chi)\end{array}}{\chi}\ \textbf{sc}$$

$$\frac{\begin{array}{l}(\phi \to \chi)\\ (\neg\phi \to \chi)\end{array}}{\chi}\ \textbf{sc}$$

Negation of biconditional (T90)

$$\frac{\neg(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \neg\psi}\ \textbf{nb}$$

$$\frac{\phi \leftrightarrow \neg\psi}{\neg(\phi \leftrightarrow \psi)}\ \textbf{nb}$$

De Morgan's laws (T63-T66)

$$\frac{(\phi \wedge \psi)}{\neg(\neg\phi \vee \neg\psi)}\ \textbf{dm}$$

$$\frac{\neg(\phi \wedge \psi)}{(\neg\phi \vee \neg\psi)}\ \textbf{dm}$$

$$\frac{\neg(\neg\phi \vee \neg\psi)}{(\phi \wedge \psi)}\ \textbf{dm}$$

$$\frac{(\neg\phi \vee \neg\psi)}{\neg(\phi \wedge \psi)}\ \textbf{dm}$$

$$\frac{(\phi \vee \psi)}{\neg(\neg\phi \wedge \neg\psi)}\ \textbf{dm}$$

$$\frac{\neg(\phi \vee \psi)}{(\neg\phi \wedge \neg\psi)}\ \textbf{dm}$$

$$\frac{\neg(\neg\phi \wedge \neg\psi)}{(\phi \vee \psi)}\ \textbf{dm}$$

$$\frac{(\neg\phi \wedge \neg\psi)}{\neg(\phi \vee \psi)}\ \textbf{dm}$$

QUANTIFIER NEGATION RULES (T203-T206)

$$\frac{\neg\forall x\phi}{\exists x\neg\phi}\ \textbf{qn} \qquad \frac{\forall x\phi}{\neg\exists x\neg\phi}\ \textbf{qn}$$

$$\frac{\neg\forall x\neg\phi}{\exists x\phi}\ \textbf{qn} \qquad \frac{\forall x\neg\phi}{\neg\exists x\phi}\ \textbf{qn}$$

$$\frac{\neg\exists x\phi}{\forall x\neg\phi}\ \textbf{qn} \qquad \frac{\exists x\phi}{\neg\forall x\neg\phi}\ \textbf{qn}$$

$$\frac{\neg\exists x\neg\phi}{\forall x\phi}\ \textbf{qn} \qquad \frac{\exists x\neg\phi}{\neg\forall x\phi}\ \textbf{qn}$$

LAWS OF ALPHABETIC VARIANCE (T231-T232)

$$\frac{\forall\alpha\phi_\alpha}{\forall\beta\phi_\beta}\ \textbf{av} \qquad \frac{\exists\alpha\phi_\alpha}{\exists\beta\phi_\beta}\ \textbf{av}$$

if ϕ_β comes from ϕ_α by proper substitution of β for α