WEEK 1 :: Homework

Using the MIU-system (see instructions) complete the following.

M1. Derive the theorem MUI.

M2. Derive the theorem MUUII.

M3. Is MUIIU a theorem? If so, derive it.

M4. Is MUUIU a theorem? If so, is there a shorter derivation of it than the shortest derivation of MUIIU?

M5. Is MU a theorem? If so, derive it; if not explain why.

WEEK 2 :: Homework

Construct L1 symbolisations for each of the following sentences.

S1. The mountain moon sets too soon.

S2. Lefty can't sing the blues.

S3. Fido is not hungry.

S4. The game is cancelled only if the field is wet.

S5. If Olivia is a logician, then Olivia doesn’t love contradictions.

S6. If Rudolf leaves the party, then Jen will be upset if she isn’t already dancing.

S7. If Frank passes only if Olivia doesn't study, then she graduates only if he doesn't.

Construct annotated derivations showing that the following arguments are valid.

1:1 P ∴ ¬¬P

1:2 P. (P→Q) ∴ Q

WEEK 3 :: Homework

Construct annotated derivations showing that the following arguments are valid.

1:3 ¬P. (Q→P) ∴ ¬Q

1:4 ¬¬(P→Q). P ∴ Q

1:5 P. (R→¬Q). (P→Q) ∴ ¬R

1:9 (R→Q). (R→¬Q) ∴ ¬R

1:10 (P→(Q→¬R)). Q ∴ (P→¬R)

1:12 (P→¬P) ∴ ¬P

1:13 (T→¬P). (¬T→¬P) ∴ ¬P

1:14 (T→(P→Q)). ¬(P→Q) ∴ ¬T

1:18 (P→(Q→R)). (P→(R→S)) ∴ (P→(Q→S))

1:19 (¬P→Q). (P→Q) ∴ Q

WEEK 4 :: Homework

Construct L2 symbolisations for each of the following sentences.

S8. Loop and Lil are parakeets.

S9. Either Olivia studies or she doesn't pass.

S10. If the train is on time, then Olivia will make her flight and Frank won’t miss the show.

S11. Neither Mark nor Jen will finish their homework unless both work faster.

Construct annotated derivations showing that the following arguments are valid.

1:34 W. ((P→W)→(R→T)) ∴ ¬T→(Q→¬R)

1:53 [T7] ∴ (((P→Q)→(P→R))→(P→(Q→R)))

2:3 (P↔¬Q)→R ∴ (¬R∧P)→Q

2:5 (S∨R)→Q. ¬(P∨¬S) ∴ ¬(P↔Q)

2:11 (P∨Q). ¬P. (T→P) ∴ (Q∧¬T)

2:15 (Q→P)→R. (¬Q∨S). ¬S ∴ (¬R→T)

2:17 (P∨R)→¬¬Q. (Q∧R)→P. R ∴ (P↔Q)

2:22 (R∨S). S→¬(Q→¬P) ∴ (P∨R)

WEEK 5 :: Homework

Construct annotated derivations showing that the following arguments are valid.

2:26 (P∨¬Q). (P→(V∧T)). ((¬V∨¬Q)→T) ∴ (R∨T)

2:27 (R∨T). (¬P↔(¬P→Q)) ∴ ((R∨S)∨(T∧Q))

2:28 (P→Q)∨(R→S) ∴ (P→S)∨(R→Q)

2:29 ¬(R↔S)↔(P→Q) ∴ (R↔¬S)↔(¬P∨Q)

2:30 (¬P∧¬Q)∨(¬¬R∧¬S). ¬(S∨Q). T→(¬S→¬R∧P) ∴ ¬T

2:31 (P∧Q)→((R∨S)∧¬(R∧S)). S→((R∧Q)∨((¬R∧¬Q)∨¬P)). (R∧Q)→S ∴ (P→¬Q)

2:33 [T24] ∴ (P∧Q) ↔ (Q∧P)

2:34 [T25] ∴ P∧(Q∧R)↔(P∧Q)∧R

2:35 [T26] ∴ ((P→Q)∧(Q→R))→(P→R)

2:36 [T27] ∴ ((P∧Q)→R)↔(P→(Q→R))

2:37 [T28] ∴ ((P∧Q)→R)↔((P∧¬R)→¬Q)

2:68 [T59] ∴ (P∨¬P)

2:74 [T65] ∴¬(P∧Q)↔(¬P∨¬Q) **[complete without using dm]**

WEEK 6 :: Homework

Construct countermodels demonstrating that the following arguments are invalid.

C1. P ∴ Q

C2. (P→Q). Q ∴ P

C3. (P→Q) ∴ (Q→P)

C4. ((P∧Q)→R) ∴ (P→R)

C5. ((P→Q)∧R). (R∨P) ∴ Q

C6. (P→Q). (¬P→R). (¬Q→¬R) ∴ P

C7. (P→Q). (¬P→R). (¬Q→¬R) ∴ R

C8. (R↔S). (T→W). (¬S∨¬Q) ∴ (¬Q∨T)

C9. ¬(P∧¬Q). P ∴ P→(Q→¬P)

C10. ¬P→(Q∨R) . R→(Q→¬P). (Q→R) ∴ ¬(P↔Q)

WEEK 7 :: Homework

Construct L3 symbolisations for each of the following sentences.

S12. Everything is blue.

S13. Something is round.

S14. All sweets are good.

S15. Some sweets are good.

S16. Some sweets aren't good.

S17. Something is round and something is square, but it is not the case that something is a round square.

Construct annotated derivations showing that the following arguments are valid.

3:0 Fa ∴ Fa ∨ Ga

3:1 ∀xFx ∴ Fa

WEEK 8 :: Homework

Construct annotated derivations showing that the following arguments are valid.

3:2 ∀x(Fx ∧ Gx) ∴ ∀xGx

3:3 ∃xFx ∴ ∃x(Gx → Fx)

3:4 ∀x(Fx→(¬Gx → Hx)) ∴ ∀x(Fx → (Gx ∨ Hx))

3:5 ∀x(Fx → Gx). ∀x(Gx → Hx) ∴ (Fa → ∃x(Gx ∧ Hx))

3:6 (∀x¬Fx → ∀xFx) ∴ ∃xFx

3:7 (∀xFx ∨ ∀xGx). ∀x(Fx → ¬Gx) ∴ ∃xFx → ∀xFx

3:8 (P → ∀x(Fx → Fa)). (∀x(Fx → Gx) ∧ ∀x(Gx → Hx)) ∴ ¬Ha → (¬P ∨ ∀x¬Fx)

3:9 ∃x(Fx ∨ Ga). ∀x(Fx → Gx) ∴ ∃xGx

3:10 ∀x(Fx → Gx) ∴ ∀x((Fx ∧ ¬∃y(Gy ∧ Hy)) → ∃x¬Hx)

3:11 ∀x(Fx↔P). ∃xFx ∴ ∀xFx

WEEK 9 :: Homework

Construct annotated derivations showing that the following arguments are valid.

3:12 ∃y∀x(Fx↔Fy). ∃xFx ∴ ∀xFx

3:13 ∀x((Fx ∧ (Gx ∨ Hx)) → Jx). ∀x((Jx ∧ Hx) → Kx). ∀x(Lx → Hx) ∴ ∀x((Fx ∧ Lx) → Kx)

3:14 ∀x(Fx ↔ (Gx ∨ Hx)). ∃xGx. ∀x(Fx → ∀xHx) ∴ ∀xFx

3:15 ∀x(Fx → Gx). ∃x((Fx ∧ Hx) ∨ (Fx ∧ Jx)) → ¬∀x(Fx → Gx) ∴ ∀x(Fx → ¬Jx)

3:16 ∃x(Fx ∧ ¬Gx) → ∀x(Fx → Hx). ∃x(Fx ∧ Jx) ∴ ∀x(Fx → ¬Hx) → ∃x(Jx ∧ Gx)

3:17 ¬∃x(Fx ∧ (Gx ∨ Hx)). ∃x(Ix ∧ Fx). ∀x(¬Hx → Jx) ∴ ∃x(Ix ∧ Jx)

3:18 ∀x(Fx → ∀xGx) ∴ ∀x(Fx → ∀x(Gx ∨ Hx))

3:19 ∃xFx → ∀xGx. ∀x(Gx ∨ Hx)→∀xJx ∴ ∀x(Fx → Jx)

3:40 ∃y∀x(Fx ↔ Fy) . ∃xFx ∴ ∀xFx

3:43 ∃y∀x(Fx ∧ Gy) ∴ ∀x∃y(Fx ∧ Gy)

3:44 T201 ∴ ∀x(Fx → Gx) → (∀xFx → ∀xGx)

3:47 T204 ∴ ¬∃xFx ↔ ∀x¬Fx

3:48 T205 ∴ ∀xFx ↔ ¬∃x¬Fx

3:49 T206 ∴ ∃xFx ↔ ¬∀x¬Fx

3:80 T238 ∴ ∀xFx → ∃xFx

3:104 ∀x(Fx ↔ (¬Gx ∨ ¬Hx)). ¬∀x(Gx ∧ Hx) → ∃x(Ix ∧ ¬Gx) ∴ ∃xFx → ∃x(Ix ∧ Fx)

3:125 ∃x(Fx → ∀xGx) ∴ ∃x∃y(¬Fx ∨ Gy)

3:129 ∴ ∀y∃x(Fy ∧ Gx) → ∃x(Gx ∧ Fx)

WEEK 10 :: Homework

Construct countermodels demonstrating that the following arguments are invalid.

C11. Fa ∴ Gb

C12. ∴ ∀x(Fx→∀xFx)

C13. (∀xFx → P) ∴ ∀x(Fx → P)

C14. ∃x(Fx → P) ∴ (∃xFx → P)

C15. ∴ ¬(∃x¬Fx ∧ (Fa ∧ Fb))

C16. ∀x∃y(Fx ↔ Gy) ∴ ∃y∀x(Fx ↔ Gy)

C17. ∃xGx. ∀x(Gx → Hx) ∴ ∀xHx

C18. Fa. Fb. ∃xFx ∴ ∀xFx

C19. ∀y(Fy → ∀xGx). ∃xFx ∴ ∀xFx

C20 ∴ ∀y∃x(Fy ∧ Gx) → ∃x(Gx ∧ ¬Fx)

C21. ∃xGx → ∀x(Gx → Hx). ∀x(Hx ∨ (Jx → Fx)) ∴ Ga → ∃xFx

For each of the following arguments either construct a derivation of the conclusion from the premises or show that it is invalid by constructing a relevant countermodel.

A1. ∃y∀x(Fx ↔ Fy). ∃xFx ∴ ∀xFx

A2. ∃y(Gy → Fy). ∃xFx ∴ (∀xFx ∨ ∃xGx)

A3. (∃x(Fx ∧ ¬Gx) → ∀x(Fx → Hx)). ∃x(Fx ∧ Jx) ∴ (∀x(Fx ∧ ¬Hx) → ∃x(Jx ∧ Gx))

A4. ∴ (∀y∃x(Fy ∧ Gx) ∨ ∃x(Gx ∧ Fx))

Input into ∃LOGIC to check: http://www.elogic.brianrabern.net/